28. A solid nonconducting sphere has a positive charge *q* spread uniformly throughout its volume. The charge density or charge per unit volume, therefore, is $\frac{q}{\frac{4}{3}\pi R^3}$. Use Gauss' law to show that the electric field at a point within the sphere at a radius *r* has a magnitude of $\frac{qr}{4\pi\varepsilon_0 R^3}$.

(Hint: For a Gaussian surface, use a sphere of radius r centered within the solid sphere of radius. Note that the net charge within any volume is the charge density times the volume.)

Gauss's Law is

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos(\theta_{EA}) = \frac{q_{enclosed}}{\varepsilon_0}$$

For a spherical charge distribution the electric field is radially in or out depending on the sign of the charge. Using a Sphere as our Gaussian surface the normal to the sphere is a radius so the angle $\theta_{EA} = 0$ and Gauss's law becomes

$$EA = \frac{q_{enclosed}}{\varepsilon_0}$$

Solving for E

$$E = \frac{q_{enclosed}}{A\varepsilon_0} = \frac{q_{enclosed}}{4\pi r^2\varepsilon_0}$$

Now $q_{enclosed}$ is the charge inside the Gaussian sphere. So that is the volume charge density times the volume enclosed. The volume charge density is just the total charge divided by the total volume. Assuming a charge Q is spread uniformly throughout a sphere of radius R.

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

SO the charge enclosed in a sphere of radius r is found from

$$q_{enclosed} = \rho V_{enclosed} = \rho\frac{4}{3}\pi r^3 = \left(\frac{3Q}{4\pi R^3}\right)\frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$$

Putting $q_{enclosed}$ into the equation for electric field yields

$$E = \frac{q_{enclosed}}{4\pi r^2\varepsilon_0} = \frac{Q\frac{r^3}{R^3}}{4\pi r^2\varepsilon_0} = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Which is what we were supposed to show that the electric field inside a spherical volume of uniformly distributed charge Q is

$$\boldsymbol{E = \frac{Qr}{4\pi\varepsilon_0 R^3}}$$

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