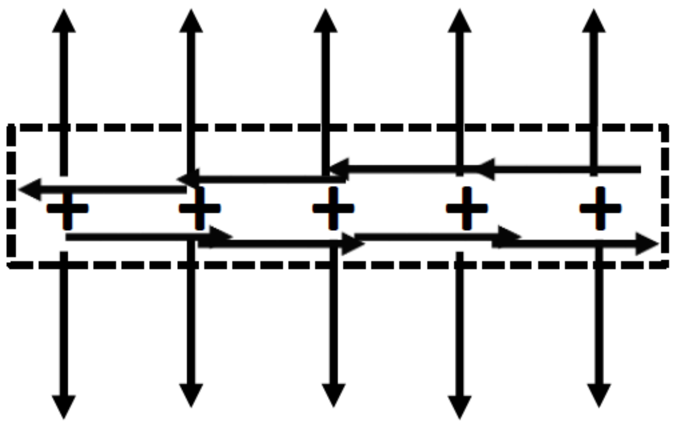
30. A long, thin, straight wire of length L has a positive charge Q distributed uniformly along it. Use Gauss' law to show that the electric field created by this wire at a radial distance r has a magnitude of $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, where $\lambda = \frac{Q}{L}$

(Hint: For a Gaussian surface, use a cylinder aligned with its axis along the wire and note that the cylinder has a flat surface at either end, as well as a curved surface.)



So the field lines for a charges can be represented as shown above. The field lines that are in directions other than radially outward from the line of charge will cancel. So the electric field lines point either radially in or out of the line of charge depending on whether the charges are positive (outward) or negative (inward). Next the Gaussian surface is a cylinder and the normal to the area of the cylinder is also radially in or out. The ends have no lines passing through so only the cylinder area not the end caps contribute to the electric flux if the line is infinitely long. So Gauss's law gives us

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos(\theta_{EA}) = \frac{q_{enclosed}}{\varepsilon_0}$$

Here the area is $A = 2\pi rL$, and again $\theta_{EA} = 0$ so E is

$$E = \frac{q_{enclosed}}{2\pi rL\varepsilon_0}$$

The charge density λ is the charge divided by the length so

$$\lambda = \frac{q}{L}$$

$$E = \frac{q_{enclosed}}{2\pi rL\varepsilon_0} = \frac{q_{enclosed}/L}{2\pi r\varepsilon_0} = \frac{\lambda}{2\pi r\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi r\varepsilon_0}}$$

As we were asked to show.

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