**Wheatstone Bridge Lab – Finding Unknown Resistances**

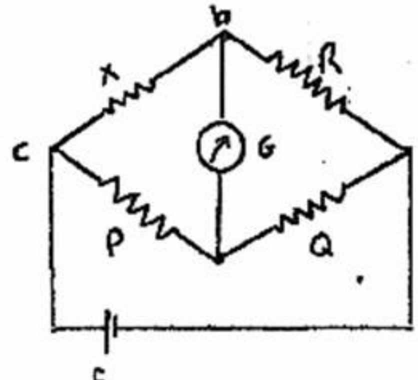



FIG. I

The circuit shown is known as a Wheatstone Bridge circuit. If the resistors R, P, and Q are known, the unknown resistance X can be found. The bridge is said to balance if the galvanometer G shows no current deflection when an emf ε is applied to the circuit. To aid in analyzing this circuit, currents have been added. $i_u$ refers to the current from points c to b to d, while $i_L$ refers to the current from c to a to d.

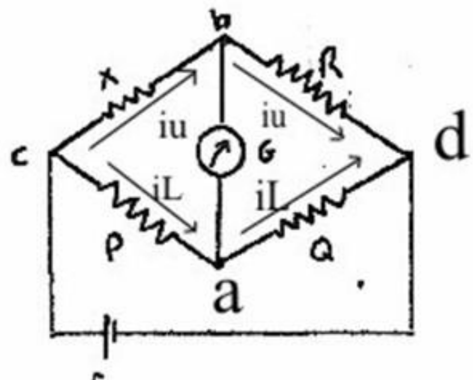



FIG. I

IF there is no current deflection in the galvanometer, then the voltage at point b above the galvanometer is equal to the voltage at point a below the galvanometer. Therefore the voltage change going from c to b is $-i_uX$ , but the voltage change going from c to point a is $-i_LP$ and these should be the same so we have the equation $-i_uX = -i_LP$. Similarly the voltage change going from b to d is $-i_uR$ and the voltage change from a to d is $-i_LQ$. Again these must be the same so we get the equation $-i_uR = -i_LQ$.

Dropping the minus signs which are everywhere we have:

$$i_u X = i_L P$$

and

$$i_u R = i_L Q$$

Rearranging these two equations:

$$\frac{i_u X}{i_L P} = 1$$

and

$$\frac{i_u R}{i_L Q} = 1$$

Setting them equal

$$\frac{i_u X}{i_L P} = 1 = \frac{i_u R}{i_L Q}$$

Canceling terms that are common on both sides we get

$$\frac{X}{P} = \frac{R}{Q}$$

So if you know R and can find P and Q you have X. To do this we shall use the following circuit:

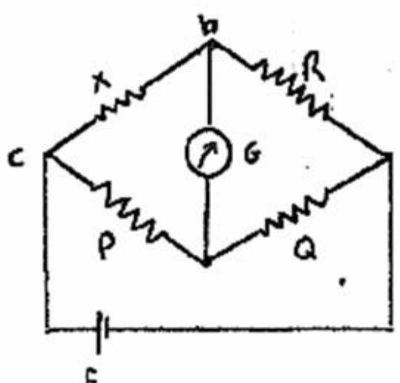



FIG. I

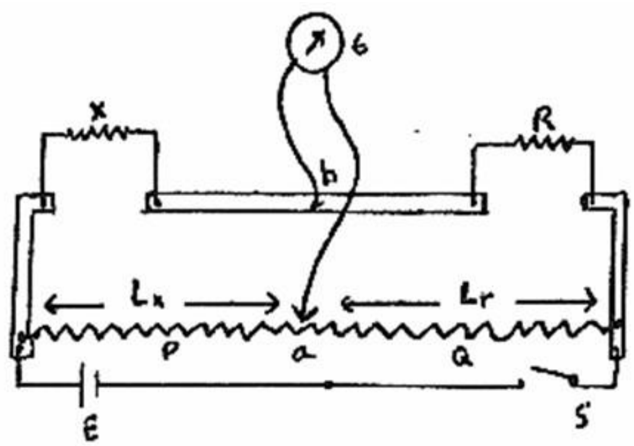



FIG. II

X will be a normal resistor similar to those you have used previously. Three resistors are provided for you to find their values. R will be a decade resistor box which has four dials that allow you to dial different resistances either from 0.1 to 1000 Ω or 1 to 10,000 Ω depending on the model of the decade resistor box. P and Q will be resistances made up from the resistance of a wire that runs the length of the apparatus. Remember that Resistance R can be found from the relation $R = \frac{\rho L}{A}$ where ρ is the resistivity, L is the length of the wire and A is the wire's cross-sectional area. Since P and Q show up as the ratio P/Q and the wire is the same, we can replace P/Q with L/(100-L) since if

P is L long, Q must be (100-L) long. So, if you dial a resistance on R to some value and then adjust the length of L until the galvanometer stops moving, you can find X from the equations:

$$X_{Left} = R\frac{L}{100 - L} \qquad X_{Right} = R\frac{100 - L}{L}$$

The procedure for this experiment proceeds best if you adjust R to some value that allows the bridge to balance when the slide touches the wire at some L when L is between 40 and 60 cm. If you are outside this range, your balance, while possible, is less accurate. So find the R, which leads to L being between 40 and 60 cm that is your rough balance of the bridge. To improve the accuracy, once you find this starting point, leave L alone and adjust R downwards to find the smallest R such that if you go any smaller the galvanometer deflects. This is your $R_{lowest}$, record that. Then adjust R upwards to find the largest R such that if you go any higher, the galvanometer deflects. This is your $R_{highest}$, record that. Now, $R_{final}$ is the average of $R_{lowest}$ and $R_{highest}$, $(1/2(R_{highest} + R_{lowest}))$ and ΔR is the difference between $R_{lowest}$ and $R_{highest}$, $(R_{highest} – R_{lowest})$

Now, you will leave R at $R_{final}$ and do a similar procedure moving L to find $L_{lowest}$ and $L_{highest}$. Then find $L_{final}$ and ΔL the same as you did with the R's.

The fact that you have a range of R's and L's that can vary over ranges ΔR and ΔL and not affect the galvanometer indicates that the accuracy of X is also limited to a range ΔX. Without proving or deriving, the formula for ΔX is given as

$$\Delta X = X\left\{\frac{\Delta R}{R} + \frac{\Delta L}{L}\left(\frac{100}{100 - L}\right)\right\}$$

There is a final systematic error that this experiment could contain. That is the wire might be damaged in some places, which would make the resistivity and/or cross-sectional area on the left, P, different from the resistivity and/or cross-sectional area on the right, Q. So, to remove this source of error, swap your unknown and your decade resistance box and repeat your experiment. Note: when swapping X and R you are also using opposite wire lengths. Now the length of wire related to X is (100-L) instead of L. That is why you have the two different equations for X depending on if it is on the left or the right. You should find only minor changes in your L's and R's upon doing this. If you find major differences, consult with your instructor.

After finding the new values average your two values for X and compare with your nominal values. Typically, resistors are known to have uncertainties in their values of 10 or 20% of the nominal value. Precision resistors with better accuracy are much more expensive.

The following are equations to help you fill out the worksheet for this lab:

$$R_{average} = \frac{1}{2}\left(R_{lowest} + R_{highest}\right)$$

$$\Delta R = R_{highest} - R_{lowest}$$

$$L_{average} = \frac{1}{2}\left(L_{lowest} + L_{highest}\right)$$

$$\Delta L = L_{highest} - L_{lowest}$$

$$X_{Left} = R_{average}\frac{L_{average}}{100 - L_{average}}$$

$$X_{Right} = R_{average}\frac{100 - L_{average}}{L_{average}}$$

$$X_{average} = \frac{1}{2}\left(X_{Left} + X_{Right}\right)$$

$$\Delta X = X\left\{\frac{\Delta R}{R_{average}} + \frac{\Delta L}{L_{average}}\left(\frac{100}{100 - L_{average}}\right)\right\}$$

$$\Delta X_{average} = \frac{1}{2}\left(\Delta X_{Left} + \Delta X_{Right}\right)$$

$$Determined\ Value = X_{average}$$

$$\%\ Diff = \frac{Nominal\ Value - Determined\ Value}{Nominal\ Value}\ x\ 100$$

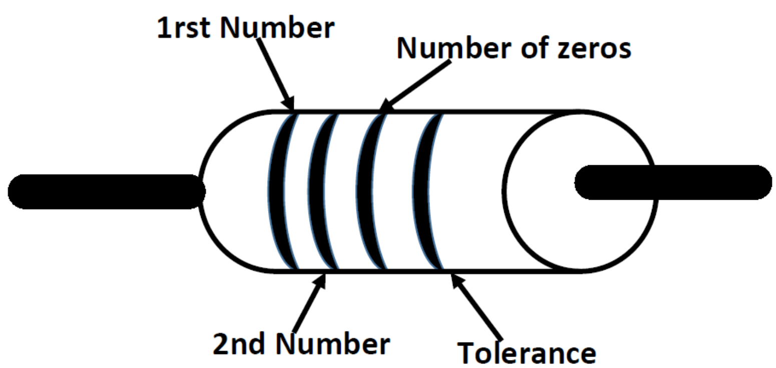
$$X_{max} = X_{average} + \Delta X_{average}$$

$$X_{min} = X_{average} - \Delta X_{average}$$

On the next page is the resistor color code and an example of how to use it.

**Resistor Color Code**

| Number | Color |
|---|---|
| 0 | Black |
| 1 | Brown |
| 2 | Red |
| 3 | Orange |
| 4 | Yellow |
| 5 | Green |
| 6 | Blue |
| 7 | Violet |
| 8 | Gray |
| 9 | White |
| Tolerance | |
| 20% | No Color |
| 10% | Silver |
| 5% | Gold |




Examples:

Colors: Brown, Black, Red, Silver

| First Number | Second Number | # of Zeros | Tolerance |
|---|---|---|---|
| Brown =1 | Black = 0 | Red = 2 | Silver = 10% |
| 1 | 0 | 00 | 10% |

So value is $1000\ \Omega\ \pm 100\ \Omega$

A resistor with a value of 6800 Ω that is 5% tolerant would have colors

Blue, Gray, Red, and Silver