Projectile Motion

A. Finding the muzzle speed v_0

The speed of the projectile as it leaves the gun can be found by firing it horizontally from a table, and measuring the horizontal range R_0. On the diagram, the y axis starts at the initial position of the projectile, and points (increases) downwards. There are two steps needed:

A.1. Find the flight time for the projectile by considering the vertical motion

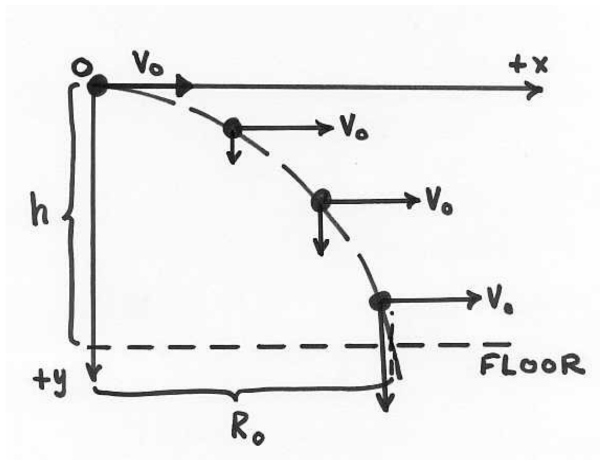
A.2. Use the flight time to calculate the horizontal distance

It is important to understand that the vertical motion is constant-acceleration motion, and the horizontal motion is constant-velocity motion (with zero acceleration).

A.1. Theory: Finding the flight time from the vertical motion

The five variables for the vertical motion are listed below. Pay careful attention to the value of v_{0y}:

$\Delta y = h$
$a_y = g$
$v_{0y} = 0\, m/s$
v_y = don't know, don't need
t = find this one



Here, $g = 9.8\, m/s^2$, and h is the height of the projectile above the floor, and both are positive.

To find the time of flight t, choose the equation that omits the unknown velocity component v_y:

$$\Delta y = v_{0y} t + \frac{1}{2} a_y\, t^2$$

In the space below, solve this for t and give the equation in terms of g and h (symbols only):

You should get:

$$t = \pm\sqrt{\frac{2\,h}{g}}$$

The negative sign refers to a time interval preceding the start of the motion, so is not relevant here (what time interval would that be?).

A.2. Theory: Finding the muzzle speed v_0 from the horizontally-fired range R_0:
For projectile motion, the horizontal component of the velocity (v_{0x}) remains constant. This means that the x coordinate of the projectile increases with time just like the position of an object moving with constant velocity: $\Delta x = v_{0x}t$. The horizontal distance traveled R_0 is called the range:

$$R_0 = v_{0x}\, t$$

When the gun is fired horizontally, there is no vertical component to the velocity of the projectile leaving the "barrel". So, $v_{0y} = 0\ m/s$. Since the initial motion is entirely horizontal, $v_{0x} = v_0$. Then the muzzle velocity comes from the above equation:

$$v_0 = \frac{R_0}{t}$$

The expression for t found in A.1 can be substituted to get an expression for the muzzle speed v_0 in terms of the horizontal range R_0, the vertical drop h, and the acceleration a_y. Do the algebra here (no numbers, just symbols):

You should have

$$v_0 = R_0 \sqrt{\frac{g}{2\,h}}$$

The actual muzzle speed of your "gun" can now be found using this equation. You will need to get a measurement of the range when the gun is fired horizontally (R_0).

A.3. Experiment: Measuring the horizontally-fired range, R_0, and finding v_0 from it:
Set the gun to fire horizontally from a table. Make sure there is plenty of space for the projectile to land without hitting anyone. Have one person "field" the projectile each time it is fired, so that it doesn't crash into the wall. After a few test shots, you should have an approximate landing area. Lightly tape a stack of paper, one sheet for each person, to the floor at this location so that you get six shots landing centrally on the page. The projectile will make an indentation where it lands, and it will be deep enough so that you can stack two or three target pages and still get a mark on all of them.

1. Before you remove the target from the floor, carefully measure the horizontal distance from the firing point to the nearest edge of the target page. Label this edge: "Near edge" and write the measured distance next to it: "dist from gun = ..."
2. Now remove the target pages
3. On your target page, carefully circle each landing point.
4. For each impact point, use a ruler to draw a line from the "near" edge to the point, and measure its length. Neatly record this data on the target page next to the line.

5. In a space on your target page, calculate the average distance of the impacts from the near edge of the page. Show your work, with units. Give the answer clearly, and also record it here:

   average distance from near edge of target page to impact points: = __________________

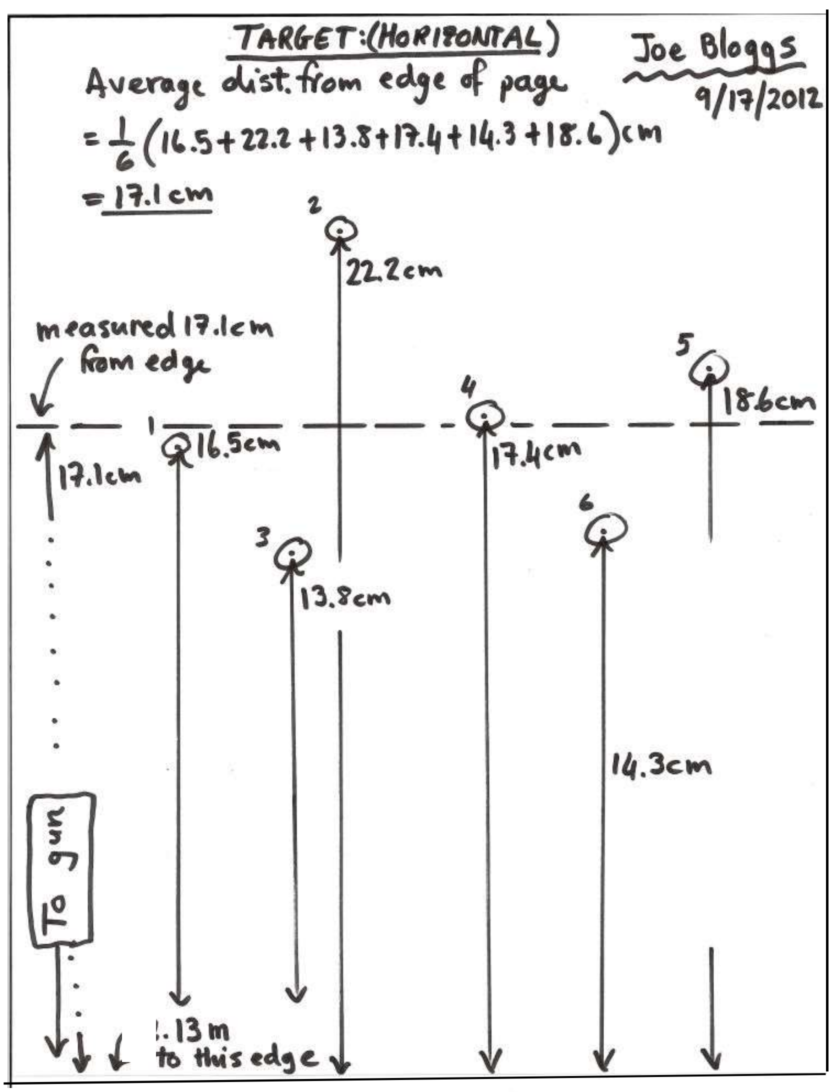
6. Find the best value for the horizontally-fired range of the projectile (for the given table height) by adding:

   R_0 = (distance from launch point to near edge of the target page)
   + (average distance of the impact points from the front edge of the page)

   = (   ) + (   )

   R_0 = ____________________________

Follow the layout of this example target sheet.



A.4. Experiment: Find the muzzle speed

To calculate the muzzle speed, the launch height above the floor is needed. Measure it accurately. Should the measurement be from the floor to the center of the ball, or the top, or the bottom? Check one, and fill in the result.

center ☐ top ☐ bottom ☐ $h =$ ________________

Use the expression from part A.2 to calculate the muzzle speed v_0 of the projectile as it leaves the "gun". Show your calculation here. First write the expression for v_0 in SYMBOLS, then substitute the numbers you have measured, then give the answer with correct unit:

$v_0 =$ ______________________

This is the projectile speed as it leaves the gun, regardless of the firing angle. You will need to use this projectile speed in section B.

Check box

NOTE: no other results from section A are carried over to section B.

B. Predicting and testing the angle-fired range $\boldsymbol{R_1}$

In this part, the idea is to use the muzzle velocity v_0 just determined to predict where the projectile will land when it is fired at a given angle. The diagram illustrates the situation, where launch angle θ will be set by the lab instructor. Note that this time we have chosen the origin of the vertical y axis on the floor and made it point (increase) upwards. The horizontal x axis has origin at the launch point and increases horizontally in the direction of fire.

Two steps are needed to calculate the predicted range:

B.1. Find the flight time t_1 ("hang time") by considering the vertical motion

B.2. Use the flight time to calculate the horizontal displacement R_1

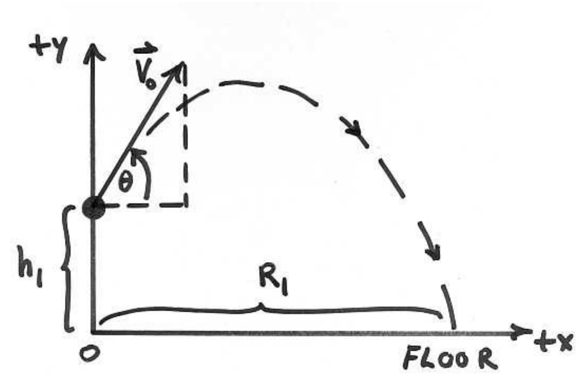
Note: t_1 and R_1 are <u>different</u> from the time and range used in part A.

B.1. Find the flight time t_1 for the angle-fired projectile

The five variables for the vertical motion are:

$\Delta y \;=\; -\,h_1$ (note: Δy is <u>negative</u>)
$a_y \;=\; -\,g$ (note: a_y is <u>negative</u>)
$v_{0y} \;=\; +\,v_o \sin\theta$
$v_y \;=$ don't know
$t \;=$ we want this; call it t_1

Here, $g = 9.8 m/s^2$, and h_1 is the height that the ball is launched above the floor.



One way to solve for t_1 is to first find the final velocity v_y and then use that to get t_1. Let's follow that method. First, look at the variables above and write down the equation that involves v_y and the other <u>known</u> quantities (symbols only):

Solve this for v_y and use the expressions on the right-hand side above to replace Δy, a_y, and v_{0y} (symbols only):

Your algebra should lead to: $v_y = \pm\sqrt{v_0^2 \sin^2\theta + 2\,g\,h_1}$

Only one of the two signs in the above equation is acceptable for the final vertical velocity at the instant just before the projectile lands on the floor. <u>Which one, and why</u>?

ANSWER: (+ / −), because ____

Place your gun on the incline and carefully measure the value of θ. No firing yet!

$\theta =$ ____________________

Measure h_1:

$h_1 =$ ____________________

Using the equation above, calculate the final vertical velocity of your projectile from the numerical values of v_o, θ, g, and h_1. Show the calculation: symbolic expression first, then the expression with substituted values, then the answer with correct sign, unit, etc.

$v_y =$ ____________________

Calculate t_1: We now have all the quantities needed to get the flight time t_1 using

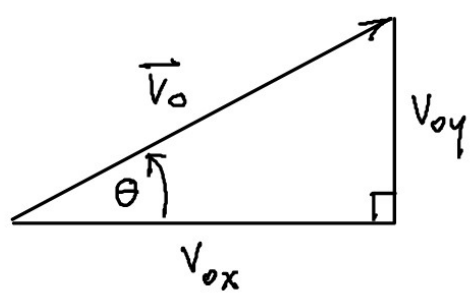
$$v_y = v_{0y} + a_y t$$

Be careful: correct v_{0y} is not the same as the muzzle velocity v_0. Show your work: first solve for the time using symbols only, and then substitute numerical quantities

$t_1 =$ ___________________________

B.2. Calculate the angle-fired range R_1 from the horizontal motion

The right-angled triangle has hypotenuse length showing the muzzle speed $v_o = |\vec{v}_0|$ found in part A. It also shows the firing angle θ, and the horizontal component of the initial velocity. With this information, find the horizontal velocity:



$v_{0x} =$

Now find the horizontal range R_1 by using $\Delta x = v_{0x} t$:

Your prediction: $R_1 =$ ______________________

Testing the predicted firing range R_1

Now that you have your predicted range R_1...

Set your gun at the angle θ following the guidelines of the instructor. Fire it a few times to identify the landing area on the floor. Then place a target sheet on the floor and follow the procedure from earlier to get about six impact points on the target sheet(s).

On the target, neatly label the distance to the near edge, circle the points, indicate with arrows the measured distances of each impact point from the near edge of the page. Neatly calculate the average distance of the points from the front edge and use this to obtain the range R_1.
Write your answer for the measured range here too:

R_1 (measured) = ________________________

Compare the measured and calculated (see B.2) ranges by finding the percentage difference:

$$\% \, diff = \left| \frac{R_1(measured) - R_1(calculated)}{R_1(calculated)} \right| \times 100$$

Show your work:

$\% \, diff$ = __________________

B.3. Another way to find the hang time for the angle-fired projectile

In B.1, we used two steps to get the flight time t_1. Another way to calculate the flight time is to use the quadratic equation

$$\Delta y = v_{0y} t + \frac{1}{2} a_y \, t^2$$

and solve for t. Do this calculation in the space provided on the hand-in sheet. You should get the same answer as you found for t_1 in part B.1.

Show all your steps: write down the quadratic formula, substitute the numbers with the correct signs and units, and then find the two solutions. The negative time corresponds to an event before the launch (what do you think this event is?), and can be ignored. The positive one should agree with your previous answer for t_1.