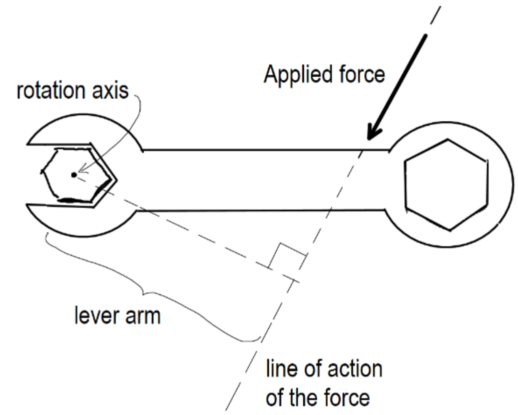
Rotational Equilibrium

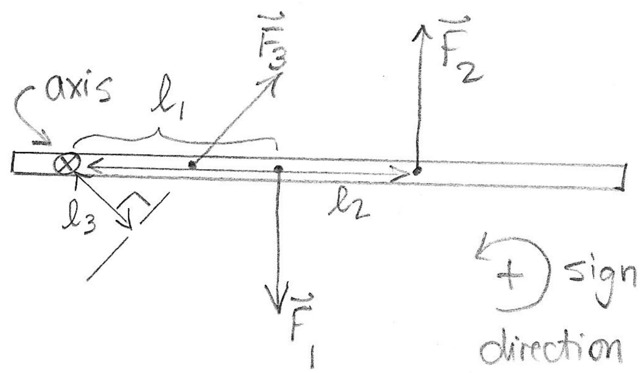
- **Torque**
  Torque, τ, is the rotational equivalent of force.
  To define it, we need an object with a chosen rotation axis. A force is applied to the object along some line of action. The lever arm, l, is the distance from the rotation axis to the line of action of the force, as shown on the diagram. So we have



$$\tau = F\,l$$

The unit of torque is Newtons × meters, or $N.m$.
Since the torque can act in either direction, we must designate the positive and negative rotation directions. For example, we might choose counterclockwise to be positive on the diagram.

- **Total torque**
  If there are several torques acting to rotate the object about one axis, the torque on the object is the sum of all the individual torques. Remember that each individual torque must have the correct sign.

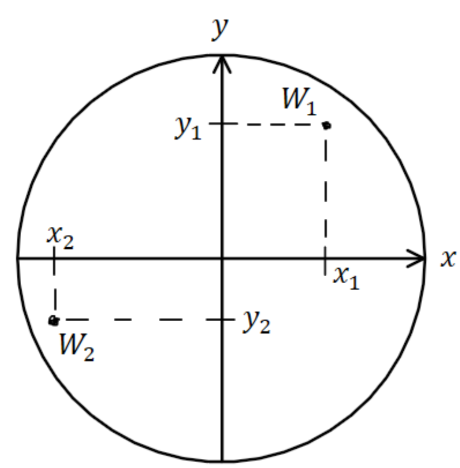


$$\Sigma\tau = F_1(-l_1) + F_2 l_2 + F_3 l_3$$

In this lab, the signs for the torques come from the signs of the coordinates of the hanging masses on the torque table. In other words, the signs are contained in the lever arm values.

- **Two dimensions**
  Using the apparatus, we will consider torques about the x axis (τ_x) and about the y axis(τ_y).
  Note that the torque about the x axis involves the y coordinates of the masses:

$$\tau_x = m_1 g\, y_1 + m_2 g\, y_2 + \ldots$$
$$\tau_y = m_1 g\, x_1 + m_2 g\, x_2 + \ldots$$



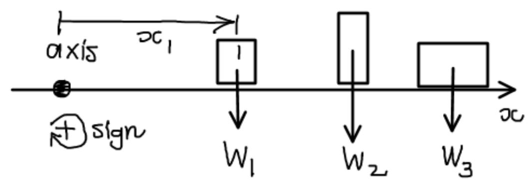
Take care to use the correct sign for each coordinate.

- **Center of mass**
For our purposes, the center of mass is the same thing as the center of gravity, defined as:

the one point at which the weight of an extended body can be considered to act when calculating the torque due to its weight

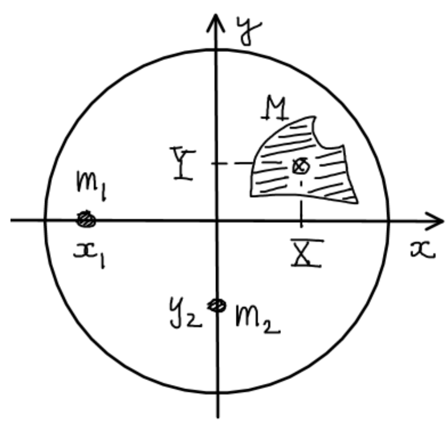
Suppose the object has three pieces, with weights W_1, W_2, W_3. This system is equivalent to a single point object with weight $W_1 + W_2 + W_3$ acting at the single point with x coordinate equal to

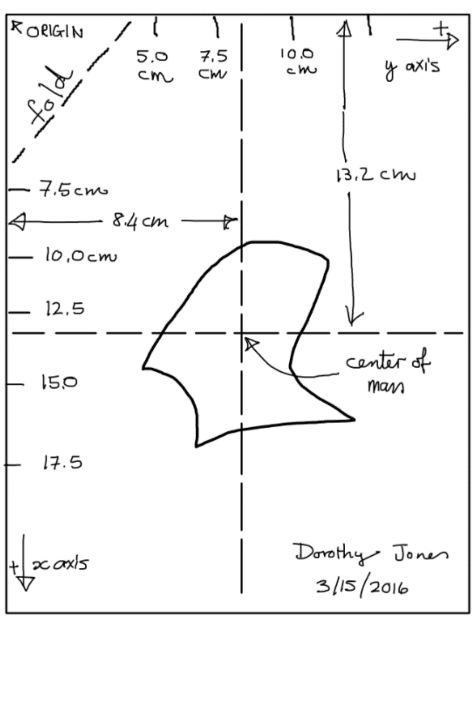
$$X_{cm} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3}{W_1 + W_2 + W_3} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$



Note: that if you cancel a factor of g from each weight you will get the usual definition of center of mass. A similar expression can be found for the y coordinate.
Note: When writing x_{cm}, the subscript *cm* means 'center of mass,' not centimeters – you can use any unit you like for x.

- **Center of mass of an irregular object (Expt 7)**
The idea is to find the center of mass of an irregularly-shaped flat aluminum object. Let m_1 = 200g, and let m_2 = 100g. Set the torque table up as shown with these two masses hanging at two different values (chosen by you) on the –x and –y axes. Place a sheet of paper on the first quadrant with edges flush against the x and y axes, and position the irregular object on top of the paper where perfect balance is achieved. (If the object is too close to the edge of the page, pick better positions for m_1 and m_2.) Trace the outline of the object, trace the 2.5 cm tick marks of the two axes, clearly show which side is x and which is y, indicate the direction in which the distance increases (use an arrow →), and write the word 'origin' at the appropriate corner of the page.





Note: Each person at the table should do this experiment, using a different orientation for the irregular shape and different x_1 and y_2 values. You can then compare the positions of the COM by stacking the outlines while holding them up to the light.

To find the coordinates (X_{cm}, Y_{cm}) of the center of mass, note that when balance is achieved, we have:

$$0 = \tau_x = M\,g\,Y_{cm} + m_2\,g\,y_2\ .$$

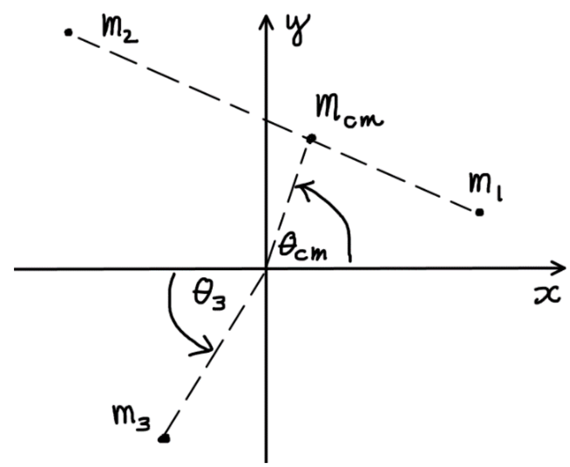
This can be solved for Y_{cm}. Do a similar calculation for X_{cm}.

Indicate the center of mass on the tracing by measuring off and drawing the two dashed lines as shown.

Center of Mass (Expt 8)

i.) Hang masses:
$m_1 = 250\ g$ at $(x, y) = (20\ cm, 10\ cm)$,
$m_2 = 150\ g$ at $(x, y) = (-17.5\ cm, 20\ cm)$.

Then take masses adding up to 240 grams, (no hanger) and position them on top of the table to balance it. As accurately as possible, read off the coordinates of this third mass, m_3, and enter these values x_3 and y_3 in the first column of the data table.



ii.) Calculate the angle θ_3 and the distance R_3 using the above results (see diagram).

iii.) Calculate the center of mass for masses 1 and 2 (exclude 3) as positioned on the table. Enter the values of x_{cm} and y_{cm} in the second column of the data table. Also calculate the angle θ_{cm} and distance R_{cm} of the center of mass.

iv.) Without moving m_3, hold the table while removing m_1 and m_2 (including the hangers). Take mass pieces of total mass $m_t = m_1 + m_2$ and position them on top of the table so as to exactly balance m_3. As accurately as possible, read the coordinates x_t and y_t of this total mass. Fill in the third column of the data table.

Consider the data table. Do you notice any numbers that are close in value? If so, is this expected, or are these just coincidences? Try to explain what you see.

Hand-in list:

1. Eight completed data tables
2. The tracing for Experiment 7, taking care not to omit any of the important information.
3. Anything else your instructor asks for.