Waves

In this lab, we are going to examine two situations, which involves waves. Properties of a wave include its **wavelength**, which is the distance between two similar points of the wave, i.e. Crest to Crest or Trough to Trough. **Frequency** is the number of complete wavelengths that pass by an observation point in a unit amount of time.

A **Node** is a point on a wave where there is zero movement. The ends of a string that is plucked on a guitar or harp would be examples of nodes. They are rigidly held in place and do not vibrate. An **Anti-Node** is a point of maximum movement of the wave. The center of the string as it is vibrating will be an anti-node.

The velocity of a wave is related to both wavelength and frequency by the following equation:

$$v = \lambda f$$

v is velocity of the wave in units similar to $^{m}/_{s}$, λ is the wavelength given in units similar to m, and f is frequency, which is found to have units of per time or the Hertz $1\ Hz = {}^{1}/_{s}$.

Standing Waves on a String Under Tension:

One situation is standing waves created on a string under tension being oscillated. The velocity of the standing waves in this case is given by the expression

$$\boldsymbol{v = \sqrt{\frac{T}{^{m}/_{L}}} = \sqrt{\frac{T}{\rho}}}$$

v is velocity of the wave in units similar to $^{m}/_{s}$, T is the tension in the string in units similar to N, m is the mass of the string in units similar to kg, L is the length of the string in units similar to m, and ρ is linear mass density in units similar to kg/m.

The velocity expression can be expressed in a power law form:

$$\boldsymbol{v = \frac{1}{\sqrt{\rho}} T^{1/2}}$$

Replacing velocity with the wavelength and frequency we can write

$$\boldsymbol{v = \frac{1}{\sqrt{\rho}} T^{1/2} = f\lambda}$$

Solving for wavelength:

$$\boldsymbol{\lambda = \frac{1}{f\sqrt{\rho}} T^{1/2}}$$

In one of the experiment's today, you have a string under tension due to weights hanging on it. By placing the proper weight on the string, you can put the standing waves into resonance, which creates either a whole number of wavelengths or an odd number of half-wavelengths.

The two ends of the string will be nodes in this apparatus. Count the number of anti-nodes and determine how many wavelengths are present. If you have just one anti-node, you have ½ a wavelength. If you have two anti-nodes, you have 1 wavelength. 5 anti-nodes would be 2 ½ wavelengths.

From the hanging weight, you can determine the tension acting on the string and by determining the number of wavelengths and using the measured length of the string between the two ends of the apparatus you can determine a wavelength. The formula is

$$\lambda = \left(\frac{2}{N}\right) L$$

Where N is the number of anti-nodes.

Since the relationship between wavelength and tension is a power law, you can do a ln-ln analysis and the slope of the plot should be close the power of tension, which is clearly $^{1}/_{2}$. The y-intercept is related to frequency and the square root of mass density. Therefore, you can calculate the frequency using the mass density, which you will measure in lab. The mass density is the mass of a piece of the string divided by its length.

For the ln-ln plot, plot ln(λ) on the vertical axis and ln(T) on the horizontal axis. Find a trend-line and print that on the plot with an equation that does not force the y-intercept to zero.

Appropriate axis labels would be

Ln(Wavelength), Ln(λ), Ln(m)
Ln(Tension), Ln(T), Ln(N)

Appropriate Graph Titles would be similar to

Ln(λ) vs. Ln(T)
D.W. Donovan August 24, 2021

Appropriate Trend-line format would be similar to

$$\mathbf{ln}(\boldsymbol{\lambda}) = \mathbf{0.456\,ln}(\boldsymbol{T}) - \mathbf{0.034}$$

Once you have the calculated frequency, you can compare it to the frequency the string was driven at by the electric oscillator and determine a percent error using the formula:

$$\%f_{error} = \frac{f_{Nominal} - f_{Calculated}}{f_{Nominal}} \; x \; 100\%$$

Standing Waves in an Air Column

A second wave situation can be created by using a tuning fork to create standing waves in a column of air. The apparatus consists of a horizontal pipe, opened at one end and closed at the other end by a membrane, which is moveable but seals the circular area of the pipe. This membrane can be moved along the pipe to create different sized columns of air.

Standing waves this time can only be an odd number of quarter of a wavelengths due to the boundary conditions that the open end of the pipe must always be an anti-node while the closed end must always be a node. So we get ¼ λ for the first anti-node, but when we add one anti-node, we must add one node, so we are always adding ½ λ each time so we see the lengths go ¼ λ, 3/4 λ, 5/4 λ, etc.

To determine the sound wavelengths generated in this experiment, you will hold a vibrating tuning fork near the open end of the tube. **NEVER LET THE TUNING FORK ACTUALLY TOUCH THE GLASS OF THE COLUMN**. You may need to remove the fork and repeatedly strike the fork and then bring it back near the open end to maintain a loud enough sound.

Start with the plunger almost to the open end of the tube making the smallest air column possible. Then slowly slide the plunger down the tube increasing the size of the air column. At first just get a general idea of where the plunger is when the sound of the tuning fork rises in loudness. Note 3 to 5 of these plunger locations. Be careful not to completely pull the plunger out of the tube.

Once you have gotten a general idea of where the 3 to 5 loud spots are, now you can go back and get as accurate a distance as possible for the very loudest point. Be sure to repeat this point several times and make sure you are hearing your set-up and not another set-up in the lab room! Fill in the data table with these distances.

Since we can only get odd number of quarter wavelengths for the one end open pipe, the relationship between length of air column and wavelength can be written as

$$L = \left(\frac{2n+1}{4}\right)\lambda$$

Where n is integers going 0, 1, 2, etc.

Now the issue is, that it is difficult to know where the 0th condition is located for a particular frequency. We can find these loud points, but it is not clear we can assign the appropriate integer to them. But so long as we can consider two consecutive loud points let's call them L_b and L_c we can subtract them as shown and we get

$$L_c - L_b = \left(\frac{2n_c + 1}{4}\right)\lambda - \left(\frac{2n_b + 1}{4}\right)\lambda = \frac{2}{4}(n_c - n_b)\lambda$$

Since the 1/4λ terms cancel out. But if they are consecutive louds, then

$$n_c - n_b = 1$$

Therefore, we have the relation

$$L_c - L_b = \frac{1}{2}\lambda$$

Or solving for wavelength

$$\boldsymbol{\lambda = 2(L_c - L_b)}$$

Now, you can find 1 less wavelengths than the number of louds you found. So if you have 3 loud points, you can get 2 wavelengths that should be close to the same size. If you have 5 louds, you can get 4 wavelengths. Average these to have just one wavelength per tuning fork.

Now using the speed of sound provided in the lab, you can solve for frequency from the basic wave velocity equation

$$f = \frac{v_{sound}}{\lambda_{average}}$$

You can of course compare with the nominal tuning fork frequency and find the % error as you did above for the other situation.

Be sure the second data table is completely filled out.