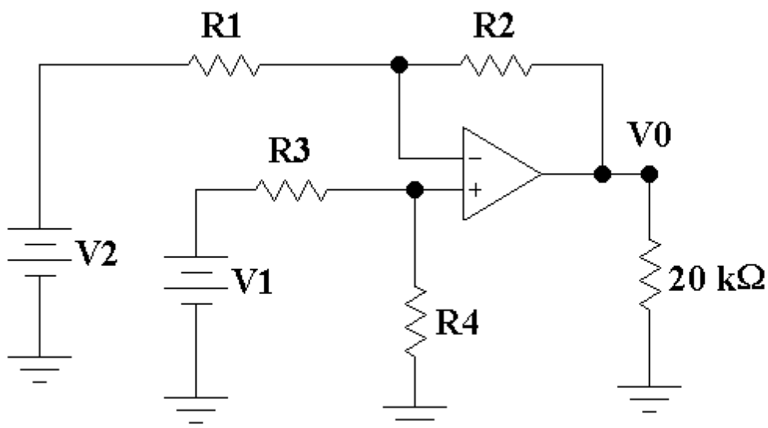
P 6.5-5 Design the operational amplifier circuit in Figure P 6.5-3 so that $v_{out} = 5 \cdot v_1 - 2 \cdot v_2$

Let's try a basic difference amplifier. The circuit is



The voltage output is given by

$$v_0 = \left(\frac{R_1 + R_2}{R_3 + R_4}\right)\left(\frac{R_4}{R_1}\right) v_1 - \left(\frac{R_2}{R_1}\right) v_2$$

Trying to simplify use

$$R_3 = R_1 \quad and \quad R_4 = R_1$$

The output reduces to

$$v_0 = \frac{R_2}{R_1}(v_1 - v_2)$$

This will not work as the coefficients need to be same and we are trying to create

$$v_0 = 5v_1 - 2v_2$$

So using the more complex relationship, make $\frac{R_2}{R_1} = 2$ This means we replace R_2 with $2R_1$

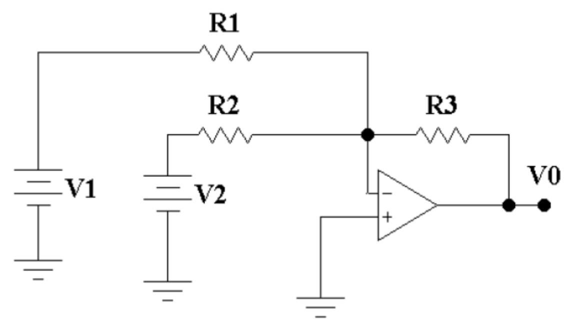
$$\left(\frac{R_1 + R_2}{R_3 + R_4}\right)\left(\frac{R_4}{R_1}\right) = \left(\frac{3R_1}{R_3 + R_4}\right)\left(\frac{R_4}{R_1}\right) = \left(\frac{3R_4}{R_3 + R_4}\right) = 5$$

$$3R_4 = 5(R_3 + R_4)$$

Which creates relation

$$R_3 = -\frac{2}{5}R_4$$

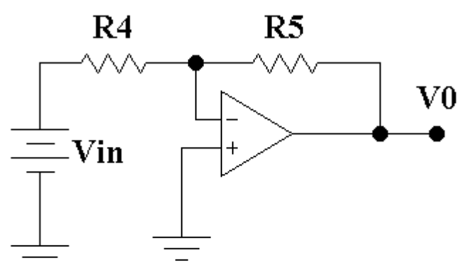
But we really don't want to deal with negative resistances, so try a new circuit. Let's consider a basic summing amplifier



The voltage output is given by

$$v_0 = -R_3\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)$$

This will allow us to produce different coefficients, but they are both negative, so if we add an inverting amplifier to one input we can swap the sign. So an inverting amplifier looks like



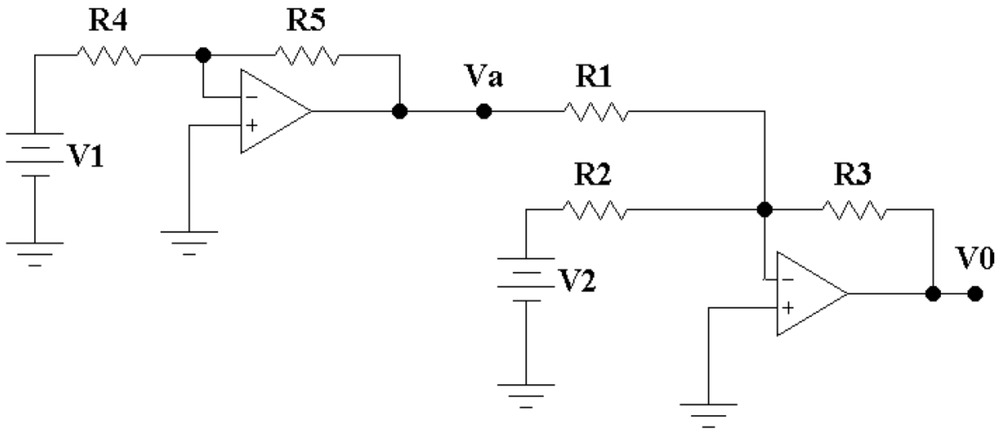
The voltage output is given by

$$v_0 = -\frac{R_5}{R_4}v_{in}$$

Since we want

$$v_0 = 5v_1 - 2v_2$$

And the summing amplifier makes things negative, we will put inverting amplifier on the first voltage and have the circuit below:



Now the voltage output is given by first consider the summing amp still gives us

$$v_0 = -R_3\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)$$

But now v_1 is v_a which is given by

$$v_a = -\frac{R_5}{R_4}v_1$$

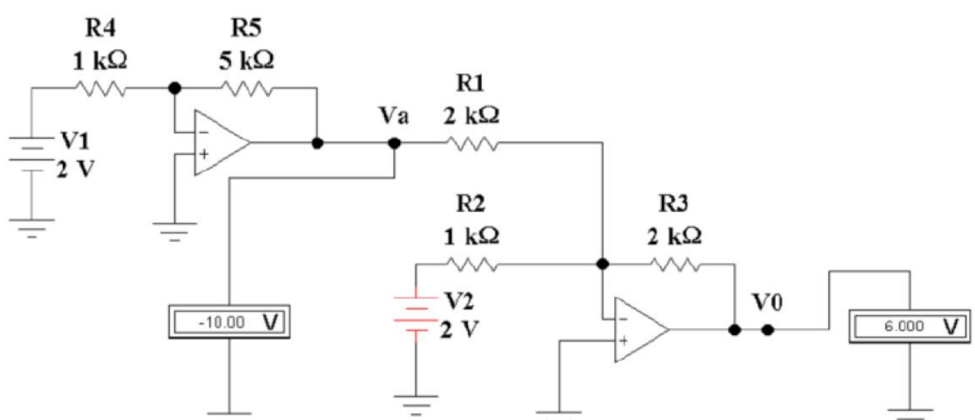
So combining we get

$$v_0 = \left(\frac{R_3R_5}{R_1R_4}\right)v_1 - \left(\frac{R_3}{R_2}\right)v_2$$

And so we must have

$$\frac{R_3R_5}{R_1R_4} = 5 \qquad and \qquad \frac{R_3}{R_2} = 2$$

Many possible choices can fulfill these conditions, so there are many possible answers, again keep the values in the kΩ range to keep op-amps acting like ideal op-amps. One choice is given by



$R_1 = 2\ k\Omega$, $R_2 = 1\ k\Omega$, $R_3 = 2\ k\Omega$, $R_4 = 1\ k\Omega$, $R_5 = 5\ k\Omega$,

As you can see if we make

$$v_1 = 2\ V, \qquad \text{and} \qquad v_2 = 2\ V$$

We correctly get a

$$v_0 = 6\ V$$

One possible answer is

$$R_1 = 2\ k\Omega$$
$$R_2 = 1\ k\Omega$$
$$R_3 = 2\ k\Omega$$
$$R_4 = 1\ k\Omega$$
$$R_5 = 5\ k\Omega$$

Other answers are possible!
As well as other circuits!!