

$v_i (V)$	$i_i (A)$
-3	-3
-4	-2
0	0
12	2
32	4
60	6

Is This element
Linear?

$$i_1 \rightarrow v_1 \quad i_1 + i_2 \rightarrow v_1 + v_2$$

$$i_2 \rightarrow v_2$$

$$2 \rightarrow 12$$

$$4 \rightarrow 32$$

$$6 \rightarrow 44 \quad 6 \rightarrow 60$$

NOT Linear!

P 2.2-1 An element has voltage v and current i as shown in Figure P 2.2-1a. Values of the current i and corresponding voltage v have been tabulated as shown in Figure P 2.2-1b. Determine if the element is linear.

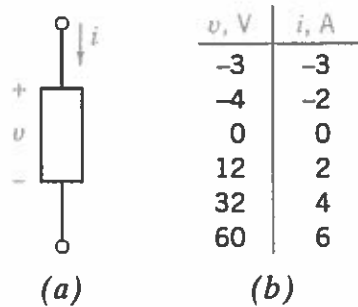


Figure P 2.2-1

An electrical element is linear if it satisfies both superposition and homogeneity. Superposition requires that a model that produces a v from an i , then consider two such points that follow:

$$\begin{aligned} i_1 &\rightarrow v_1 \\ i_2 &\rightarrow v_2 \\ i_1 + i_2 &\rightarrow v_1 + v_2 \end{aligned}$$

Homogeneity requires that a model that produces a v from an i , then consider the following:

$$\begin{aligned} i &\rightarrow v \\ ki &\rightarrow kv \end{aligned}$$

Choose two points, let's use (v,i) (12, 2) and (32,4) I choose this to make the homogeneity condition easier since $2(2)$ is 4, so we can make $k = 2$.

Superposition

$$\begin{aligned} 2 &\rightarrow 12 \\ 4 &\rightarrow 32 \\ 2 + 4 = 6 &\rightarrow 12 + 32 = 44 \end{aligned}$$

However, we have the point(60, 6) so $6 \rightarrow 44$ is not $6 \rightarrow 60$
So Superposition fails.

Homogeneity

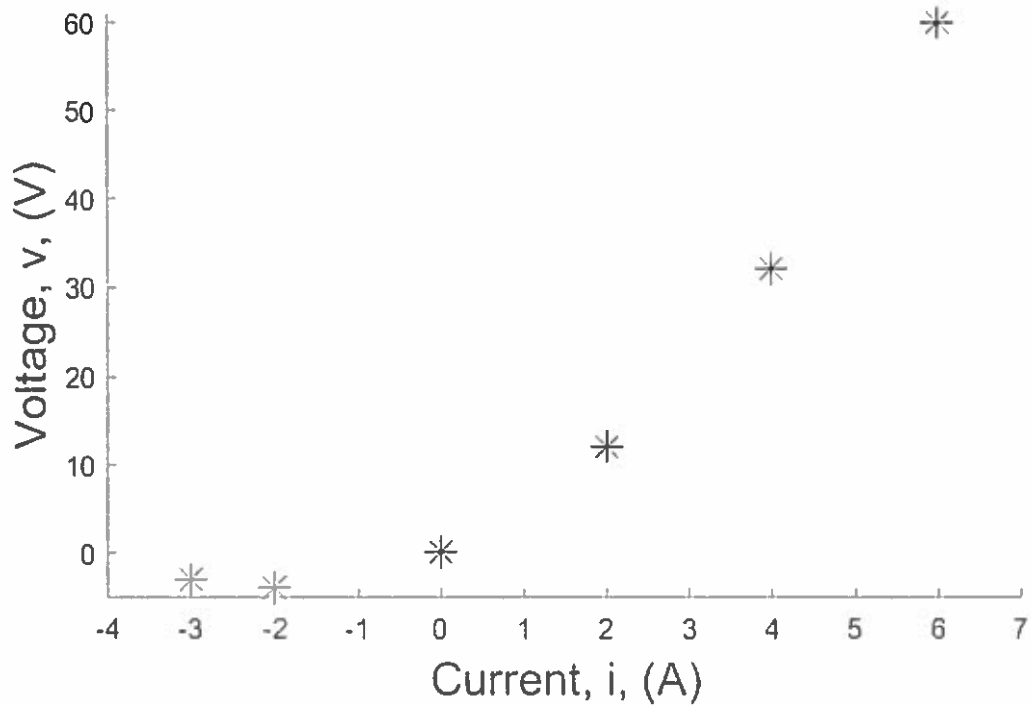
$$\begin{aligned} 2 &\rightarrow 12 \\ 4 &\rightarrow 32 \end{aligned}$$

But 4 should have gone to 24 since k would be 2. So Homogeneity fails also.

This element is not linear!

Finally, using MATLAB we can plot these points and see clearly the function is not linear.

PH 320 Homework Problem 2.2-1
Voltage vs Current for a Circuit Element
D.W. Donovan -- 27-Dec-2018



Clearly this plot is not linear!

```
%Program to Plot Element Properties in Problem 2.2-1 to  
determine  
%if the element is a linear element.  
%Version 2018-12-27 D.W. Donovan
```

```
clear all;  
v = [-3 -4 0 12 32 60]';  
i = [-3 -2 0 2 4 6]';  
x = i;  
y = v;
```

```
figure  
hold on  
plot(i, v, 'k *', 'MarkerSize', 12)
```

```
tt1 = 'PH 320 Homework Problem 2.2-1';  
tt2 = 'Voltage vs Current for a Circuit Element';
```

```

ttn = 'D.W. Donovan -- ';
tnl = '\newline';
ttf = [tt1 tnl tt2 tnl ttn date];
xl = 'Current, i, (A)';
yl = 'Voltage, v, (V)';

sp = 1;
axxmin = min(x) - sp;
axxmax = max(x) + sp;
axymin = min(y) - sp;
axymax = max(y) + sp;

title (ttf, 'FontSize', 16)
xlabel(xl, 'FontSize', 16)
ylabel(yl, 'FontSize', 16)
axis([axxmin axxmax axymin axymax])

```

**[Dr. Donovan's Classes
Page](#)**

**[Dr. Donovan's PH 320
Homework Page](#)**

**[PH 320 Homework
Chapter 2 Page](#)**

**[Dr. Donovan's Main
Web Page](#)**

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This page last updated on December 27, 2018

$\downarrow \hat{i}$	V_i (V)	\hat{i}_i (A)
+	3.078	12
\sim	5.13	20
-	12.825	50

MAT Lab Gives

$$\underline{V = 0.2565 \hat{i} + 4.5983 \times 10^{-15} \hat{i}}$$

Superposition

$$\begin{aligned} \hat{i}_1 &\rightarrow V_1 = 0.2565 \hat{i}_1 \\ \hat{i}_2 &\rightarrow V_2 = 0.2565 \hat{i}_2 \\ \hat{i}_1 + \hat{i}_2 &\rightarrow 0.2565 \hat{i}_1 + 0.2565 \hat{i}_2 \\ &= 0.2565 (\hat{i}_1 + \hat{i}_2) \checkmark \end{aligned}$$

Homogeneity

$$i_0 \rightarrow v = 0,2565 \dot{i}$$

$$k\dot{i} \rightarrow k\cancel{v} = 0,2565 (k\dot{i})$$

$$= k (0,2565 \dot{i})$$

$$= k v$$

P 2.2-3 A linear element has voltage v and current i as shown in Figure P 2.2-3a. Values of the current i and corresponding voltage v have been tabulated as shown in Figure P 2.2-3b. Represent the element by an equation that expresses v as a function of i . This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of v corresponding to a current of $i = 6$ mA. (c) Use the model to predict the value of i corresponding to a voltage of $v = 12$ V.

Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.

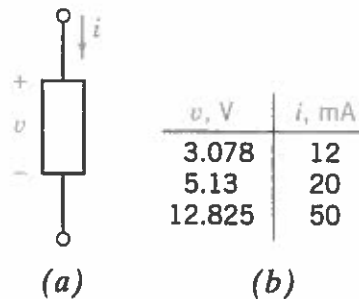


Figure P 2.2-3

An electrical element is linear if it satisfies both superposition and homogeneity. Superposition requires that a model that produces a v from an i , then consider two such points that follow:

$$\begin{aligned} i_1 &\rightarrow v_1 \\ i_2 &\rightarrow v_2 \\ i_1 + i_2 &\rightarrow v_1 + v_2 \end{aligned}$$

Homogeneity requires that a model that produces a v from an i , then consider the following:

$$\begin{aligned} i &\rightarrow v \\ ki &\rightarrow kv \end{aligned}$$

To test this since there are no obvious points that are related, let's plot this in MATLAB and get a "best straight-line" and then we will have the model to test linearity on.

From MATLAB, we find a relationship

$$v = 0.2565 i + 0$$

MATLAB found a slope of 0.2565 (V/mA) and a y-intercept of 4.5983×10^{-15} V which is zero!

(a) Verify that the model is linear.

Superposition gives us

$$\begin{aligned} i_1 &\rightarrow v_1 = 0.2565 i_1 \\ i_2 &\rightarrow v_2 = 0.2565 i_2 \\ i_1 + i_2 &\rightarrow v_1 + v_2 = 0.2565 i_1 + 0.2565 i_2 = 0.2565 (i_1 + i_2) \end{aligned}$$

Superposition is valid!

Homogeneity gives us

$$i \rightarrow v = 0.2565 i$$
$$ki \rightarrow kv = k(0.2565)i = (0.2565)ki$$

Homogeneity is valid!

Model is Linear!

(b) Use the model to predict the value of v corresponding to a current of $i = 4 \text{ mA}$.

From MATLAB we find v from $v = 0.2565 i$

$$v(i = 6 \text{ mA}) = 1.539 \text{ V}$$

(c) Use the model to predict the value of i corresponding to a voltage of $v = 12 \text{ V}$.

And again using MATLAB we solved $i = \frac{v}{0.2565}$

$$i(v = 12 \text{ V}) = 46.78 \text{ mA}$$

Model is linear:

$$v = 0.2565 \left(\frac{\text{V}}{\text{mA}}\right) i$$

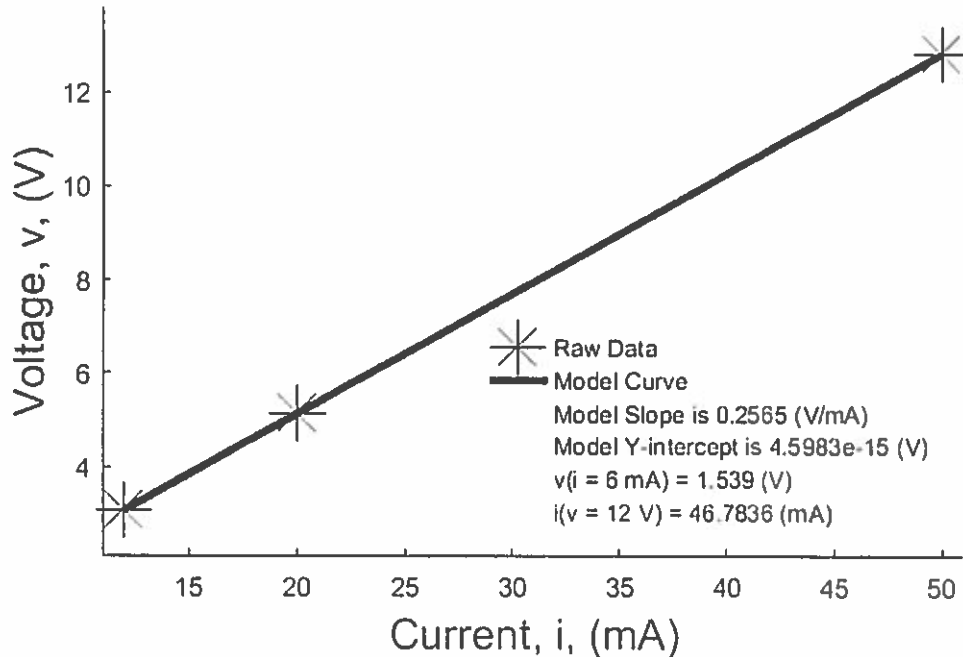
Answers for (b) and (c)

$$v(i = 6 \text{ mA}) = 1.539 \text{ V}$$

$$i(v = 12 \text{ V}) = 46.78 \text{ mA}$$

MATLAB Plot and Code follows:

PH 320 Homework Problem 2.2-3
Voltage vs Current for a Circuit Element
D.W. Donovan -- 19-Feb-2019



```
%Program to Plot Element Properties in Problem 2.2-3 to
determine
%if the element is a linear element. We can also
determine a
%mathematical model for the element.
%Version 2019-2-19 D.W. Donovan
```

```
clear all;
v = [3.078 5.13 12.825]';
i = [12 20 50]';
x = i;
y = v;
a(:,2) = i;
a(:,1) = ones(size(i,2));
c = a\v;
b = c(1);
m = c(2);

inew = [min(i) : (max(i) - min(i))/1000 : max(i)]';
vnew = m*inew +b;
vb = m*(6) +b;
```

```

ic = (12-b)/m;

ls = ['Model Slope is ' num2str(m) ' (V/mA)'];
ly = ['Model Y-intercept is ' num2str(b) ' (V)'];
lv = ['v(i = 6 mA) = ' num2str(vb) ' (V)'];
li = ['i(v = 12 V) = ' num2str(ic) ' (mA)'];

ans = {'Slope ' m; 'Y-intercept ' b;
       'v(i = 6 mA) (V) ' vb; 'i(v = 4 V) (mA) ' ic};

ans

figure
hold on
plot(i, v, 'k*', 'MarkerSize', 20)
plot(inew, vnew, 'k-', 'LineWidth', 3)
plot(max(i), min(v), '.w')
plot(max(i), min(v), '.w')
plot(max(i), min(v), '.w')
plot(max(i), min(v), '.w')

tt1 = 'PH 320 Homework Problem 2.2-3';
tt2 = 'Voltage vs Current for a Circuit Element';
ttn = 'D.W. Donovan -- ';
tnl = '\newline';
ttf = [tt1 tnl tt2 tnl ttn date];
xl = 'Current, i, (mA)';
yl = 'Voltage, v, (V)';

sp = 1;
axxmin = min(x)-sp;
axxmax = max(x) + sp;
axymin = min(y) - sp;
axyymax = max(y) + sp;

title (ttf, 'FontSize', 16)
xlabel(xl, 'FontSize', 16)
ylabel(yl, 'FontSize', 16)
legend ('Raw Data', 'Model Curve', ls, ly, lv,
li, 'Location', 'SE')
legend ('boxoff')

```

```
axis([axxmin axxmax axymin axymax])
```

```
%{  
ans = 'Slope ' [ 0.2565]  
      'Y-intercept ' [4.5983e-15]  
      'v(i = 6 mA) (V) ' [ 1.5390]  
      'i(v = 4 V) (mA) ' [ 46.7836]  
%}
```

[Dr. Donovan's Classes
Page](#)

[Dr. Donovan's PH 320
Homework Page](#)

[PH 320 Homework
Chapter 2 Page](#)

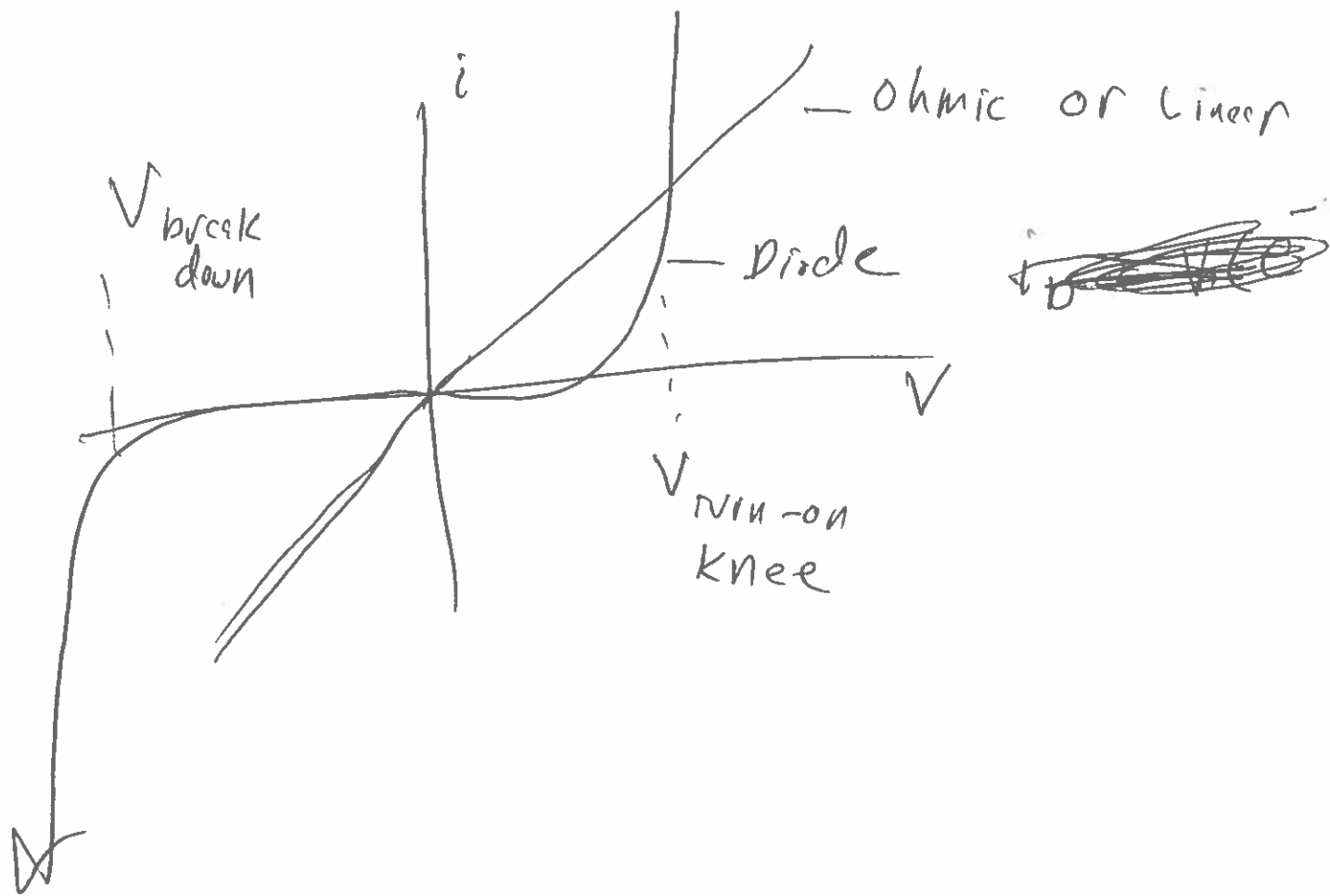
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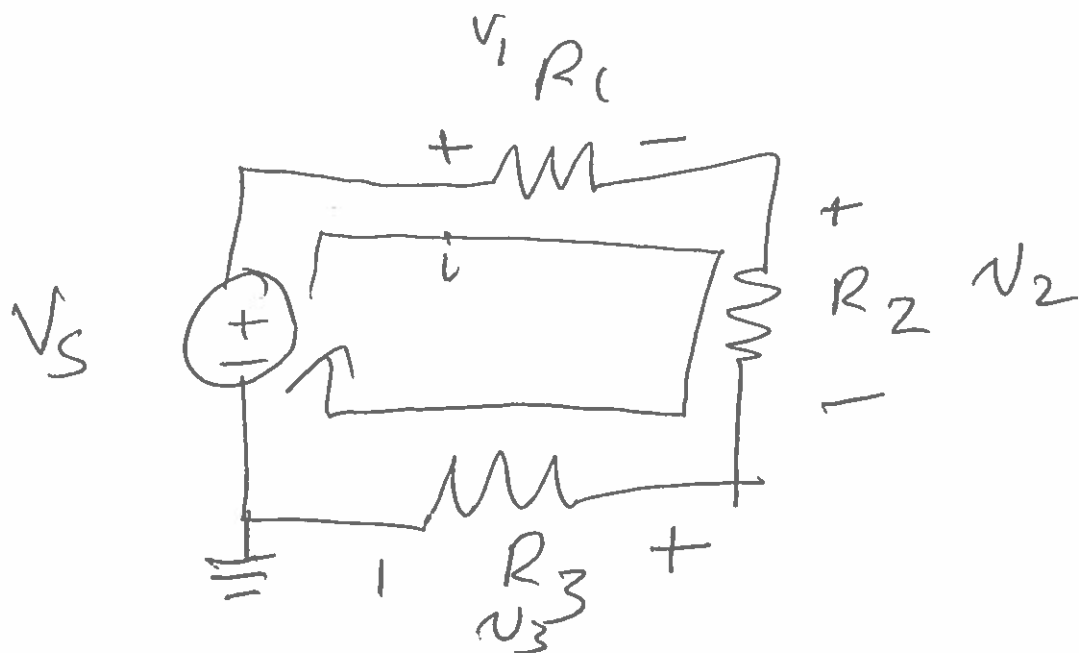
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This page last updated on February 19, 2019





~~Series~~ elements are wired in series if there is no alternative path for current other than from 1 element to the next element.

Series elements have same current.

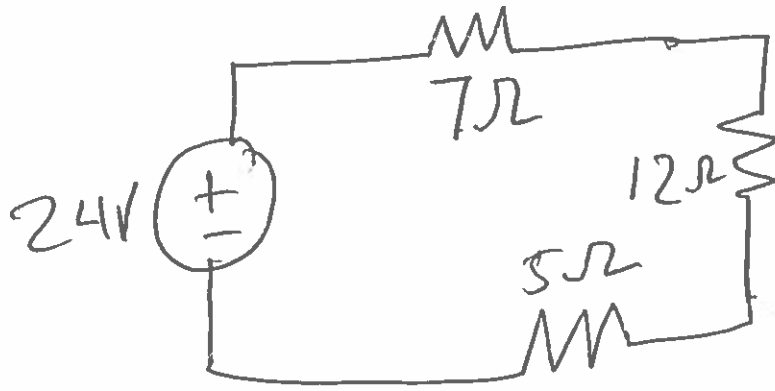
$$V_S - V_1 - V_2 - V_3 = 0$$

$$V_S = V_1 + V_2 + V_3$$

$$\hat{i}_S R_S = \hat{i}_1 R_1 + \hat{i}_2 R_2 + \hat{i}_3 R_3$$

BUT series $\Rightarrow \hat{i}_S = \hat{i}_1 = \hat{i}_2 = \hat{i}_3 = \hat{i}$

$$R_S = R_1 + R_2 + R_3 \quad \text{— Resistors in Series}$$



$$i = ? = \frac{V_S}{R_{TOT}} = \frac{24V}{7\Omega + 12\Omega + 5\Omega}$$

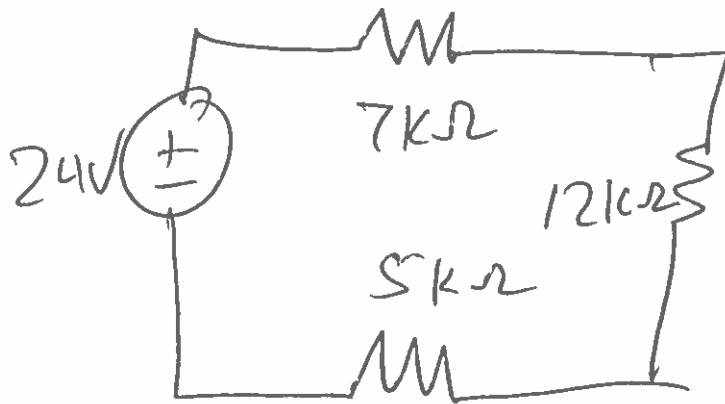
Voltage Divider

$$i = \frac{24V}{24\Omega} = 1A$$

$$V_{7\Omega} = i R_{7\Omega} = (1A)(7\Omega) = 7V$$

$$V_{12\Omega} = i R_{12\Omega} = (1A)(12\Omega) = 12V$$

$$V_{5\Omega} = i R_{5\Omega} = (1A)(5\Omega) = 5V$$



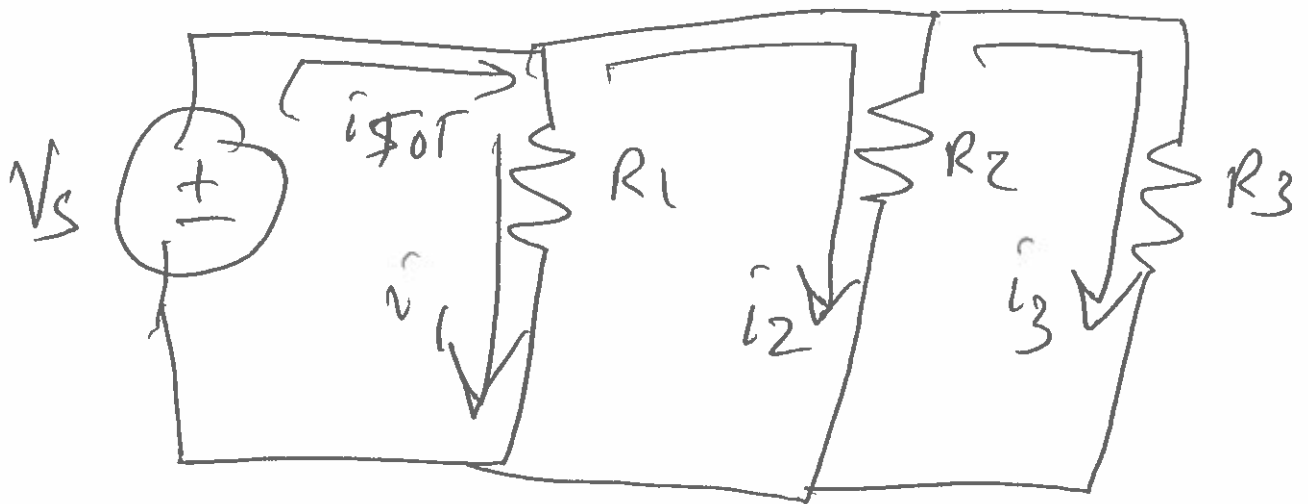
$$I \Rightarrow \frac{24V}{24k\Omega} = 1 \text{ mA}$$

A \Rightarrow Power space heaters
cooking appliances

mA \Rightarrow Power electronics, computers,
communications

Two elements are connected in parallel if both ends share the same potential difference (ie usually both ends connected)

elements in parallel have same V_{across}



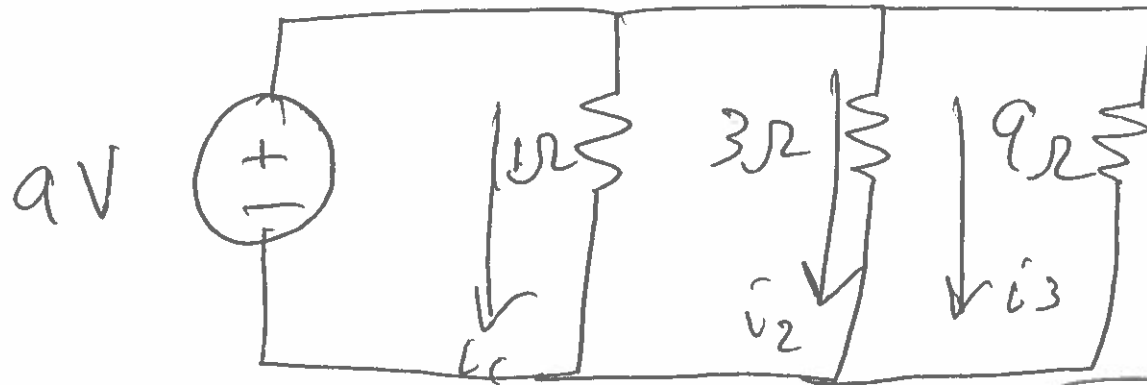
$$V_s = V_{R_1} = V_{R_2} = V_{R_3}$$

$$i_{TOT} = i_1 + i_2 + i_3$$

$$\frac{V_s}{R_{TOT}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{1}{R_{\text{TOT}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors
in Parallel



$$i_1 = \frac{9V}{1\Omega} = 9A$$

$$i_2 = \frac{9V}{3\Omega} = 3A$$

$$i_3 = \frac{9V}{9\Omega} = 1A$$

$$13A$$

Current
Divider



$$V_{8k\Omega} = ?$$

Use Voltage Division

$$V_{8k\Omega} = V_S \frac{R_{8k\Omega}}{R_{TOT}}$$

$$V_{8k\Omega} = (10V) \frac{(8k\Omega)}{(8k\Omega + 7k\Omega + 5k\Omega)}$$

$$= 10V \frac{8}{20} = \boxed{4V}$$

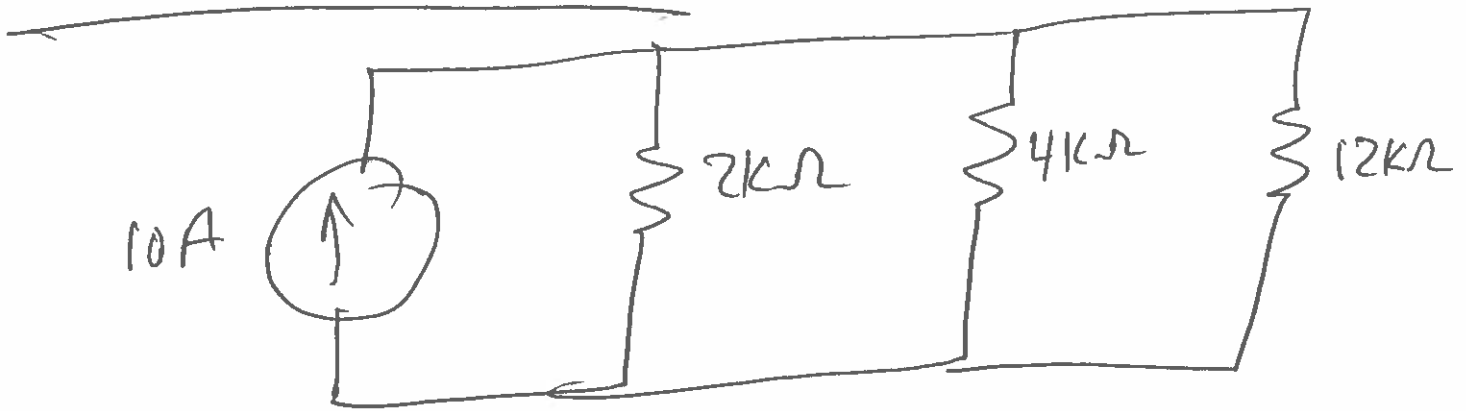
Similarly

$$V_{7k\Omega} = 3.5V$$

$$V_{5k\Omega} = 2.5V$$

} $\Rightarrow 10V$

CURRENT DIVISION



$$\hat{I}_x = \hat{I}_s \frac{\frac{1}{R_x}}{\frac{1}{R_{TOT}}}$$

$$\hat{I}_x = (10A) \left(\frac{1}{2k\Omega} \right) = \frac{10A / 2k\Omega}{\frac{1}{2k\Omega} + \frac{1}{4k\Omega} + \frac{1}{12k\Omega}} = \frac{6 + 3 + 1}{12k\Omega}$$

$$= \frac{10A}{2k\Omega} \left(\frac{12k\Omega}{10} \right) = \underline{\underline{6A}}$$