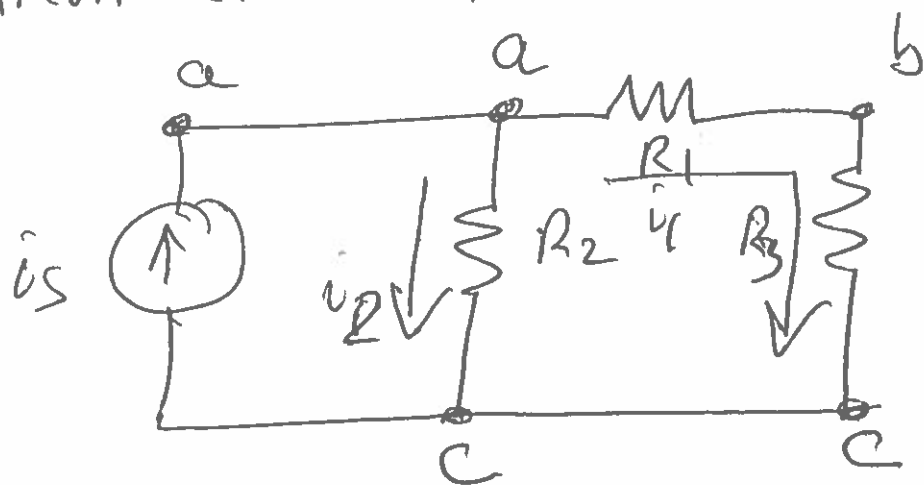


In reality both are specialized ways of  
 doing Kirchhoff equations,  
 but you do less thinking and they are very  
 easy to put into MATLAB.

Voltage nodes are points between two or  
 more circuit elements.

Consider



$V_c = 0 \Rightarrow$  Ground point

at a  $i_s = i_1 + i_2$

$$i_1 = \frac{V_a - V_b}{R_1} = \frac{V_b}{R_3} \quad i_2 = \frac{V_a}{R_2}$$

$$i_s = \frac{V_a - V_b}{R_1} + \frac{V_a}{R_2}$$

at node b

$$i_1 = i_1$$

$$\frac{V_a - V_b}{R_1} = \frac{V_b}{R_3}$$

$$i_s = V_a \left( \frac{1}{R_1} + \frac{1}{R_2} \right) - V_b \left( \frac{1}{R_1} \right)$$

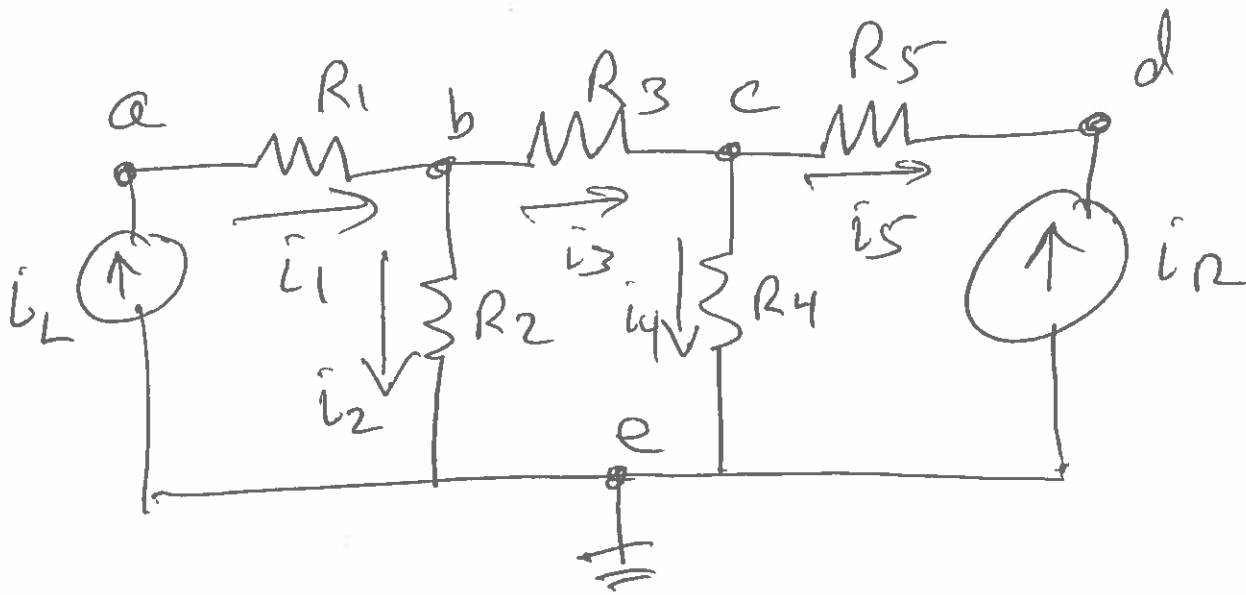
$$0 = -V_a \left( \frac{1}{R_1} \right) + V_b \left( \frac{1}{R_1} + \frac{1}{R_3} \right)$$

Plug in values for  $R_1, R_2, R_3$ , and  $i_s$

Get  $V_a, V_b$

$$\left[ \frac{1}{R} \right] [V] = [i]$$

$$V = \left[ \frac{1}{R} \right] \setminus [i]$$



at a

$$i_L = v_a \left( \frac{1}{R_1} \right) - v_b \left( \frac{1}{R_1} \right)$$

at b

$$0 = v_b \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - v_a \left( \frac{1}{R_1} \right) - v_c \left( \frac{1}{R_3} \right)$$

at c

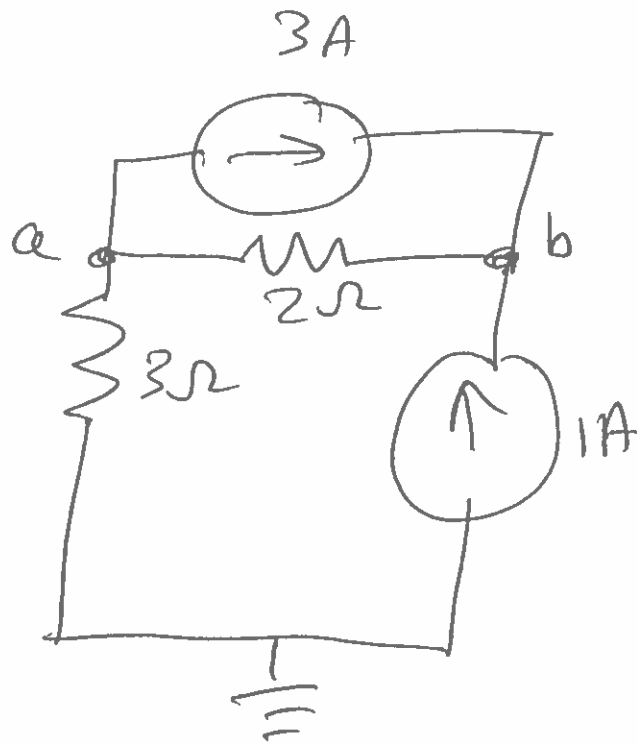
$$0 = v_c \left( \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right) - v_b \left( \frac{1}{R_3} \right) - v_d \left( \frac{1}{R_5} \right)$$

at d

$$i_R = v_d \left( \frac{1}{R_5} \right) - v_c \left( \frac{1}{R_5} \right)$$

# Voltage Node example 1

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$V_a, V_b$  ?

Node a

$$-3A = V_a \left( \frac{1}{2\Omega} + \frac{1}{3\Omega} \right) - V_b \left( \frac{1}{2\Omega} \right)$$

Node b

$$3A + 1A = V_b \left( \frac{1}{2\Omega} \right) - V_a \left( \frac{1}{2\Omega} \right)$$

$$-3A = V_a \left( \frac{1}{2\Omega} + \frac{1}{3\Omega} \right) - V_b \left( \frac{1}{2\Omega} \right)$$

$$4A = -V_a \left( \frac{1}{2\Omega} \right) + V_b \left( \frac{1}{2\Omega} \right)$$

qdd 2 above eqns

$$1A = V_a \left(\frac{1}{3\Omega}\right) \Rightarrow V_a = 3\Omega(1A)$$

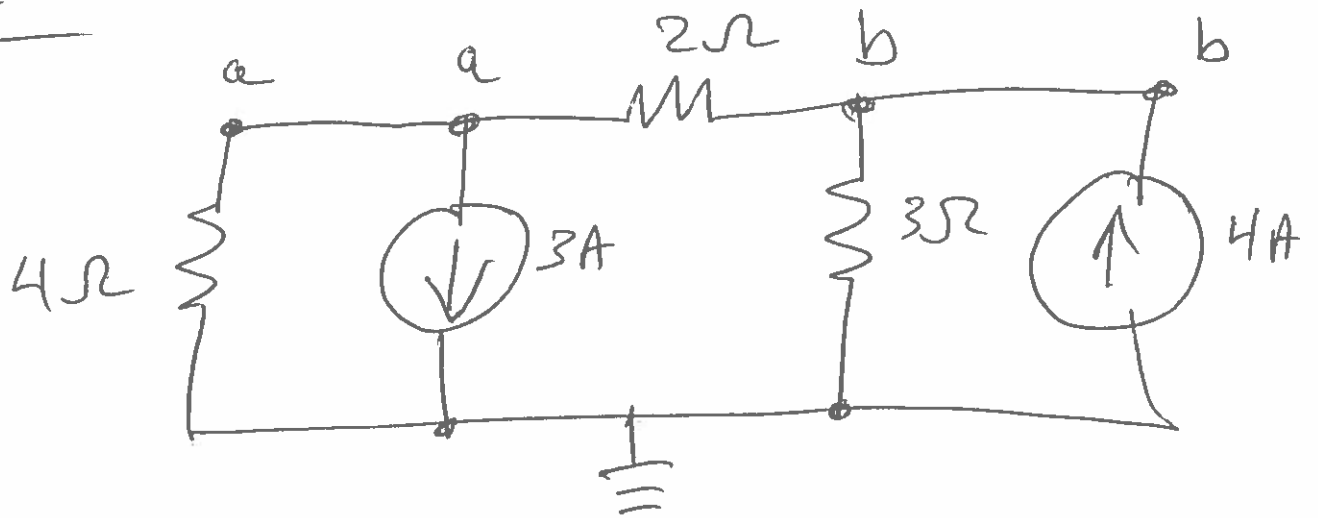
$$\boxed{V_a = 3V}$$

$$4A = V_b \left(\frac{1}{2\Omega}\right) - V_a \left(\frac{1}{2\Omega}\right)$$

$$8V = V_b - V_a = V_b - 3V$$

$$\boxed{V_b = 11V}$$

ex 2



$$a/ \quad -3A = N_a \left( \frac{1}{4\Omega} + \frac{1}{2\Omega} \right) - N_b \left( \frac{1}{2\Omega} \right)$$

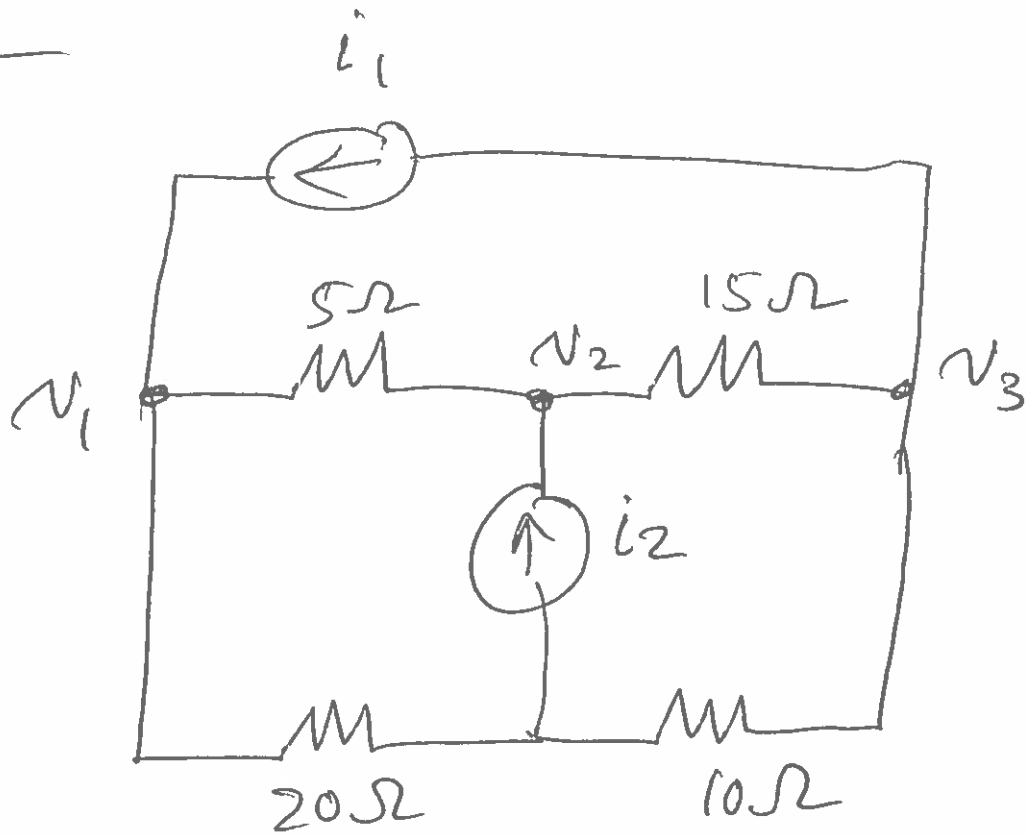
$$b/ \quad 4A = N_b \left( \frac{1}{2\Omega} + \frac{1}{3\Omega} \right) - N_a \left( \frac{1}{2\Omega} \right)$$

$$\frac{1}{R} = G \quad G = \begin{bmatrix} (1/2 + 1/4) & -(1/2) \\ -(1/2) & (1/2 + 1/3) \end{bmatrix}$$

$$\hat{i} = \begin{bmatrix} -3 & 4 \end{bmatrix}'$$

$$V = G \setminus \hat{i} \quad \Rightarrow \quad N_a = -\frac{4}{3}V \\ N_b = +4V$$

EX 3



$$v_1 = 4V, \quad v_2 = 15V, \quad v_3 = 18V$$

$$i_1 = ? \quad i_2 = ?$$

$$v_1 / i_1 = v_1 \left( \frac{1}{20\Omega} + \frac{1}{5\Omega} \right) - v_2 \left( \frac{1}{5\Omega} \right)$$

$$v_2 / i_2 = v_2 \left( \frac{1}{15\Omega} + \frac{1}{5\Omega} \right) - v_3 \left( \frac{1}{15\Omega} \right) - v_1 \left( \frac{1}{5\Omega} \right)$$

$$N_3 \quad -i_1 = N_3 \left( \frac{1}{15\Omega} + \frac{1}{10\Omega} \right) - N_2 \left( \frac{1}{15\Omega} \right)$$

Use  $V_1$   
to solve  
for  $i_1$

$$i_1 = 4V \left( \frac{1}{20\Omega} + \frac{1}{5\Omega} \right) - 15V \left( \frac{1}{15\Omega} \right)$$

$$\frac{1+4}{20\Omega} = \frac{5}{20} = \frac{1}{4} \quad 3A$$

$$i_1 = 1 - 3A = \boxed{-2A = i_1}$$

Use  $V_2$   
to solve  
for  $i_2$

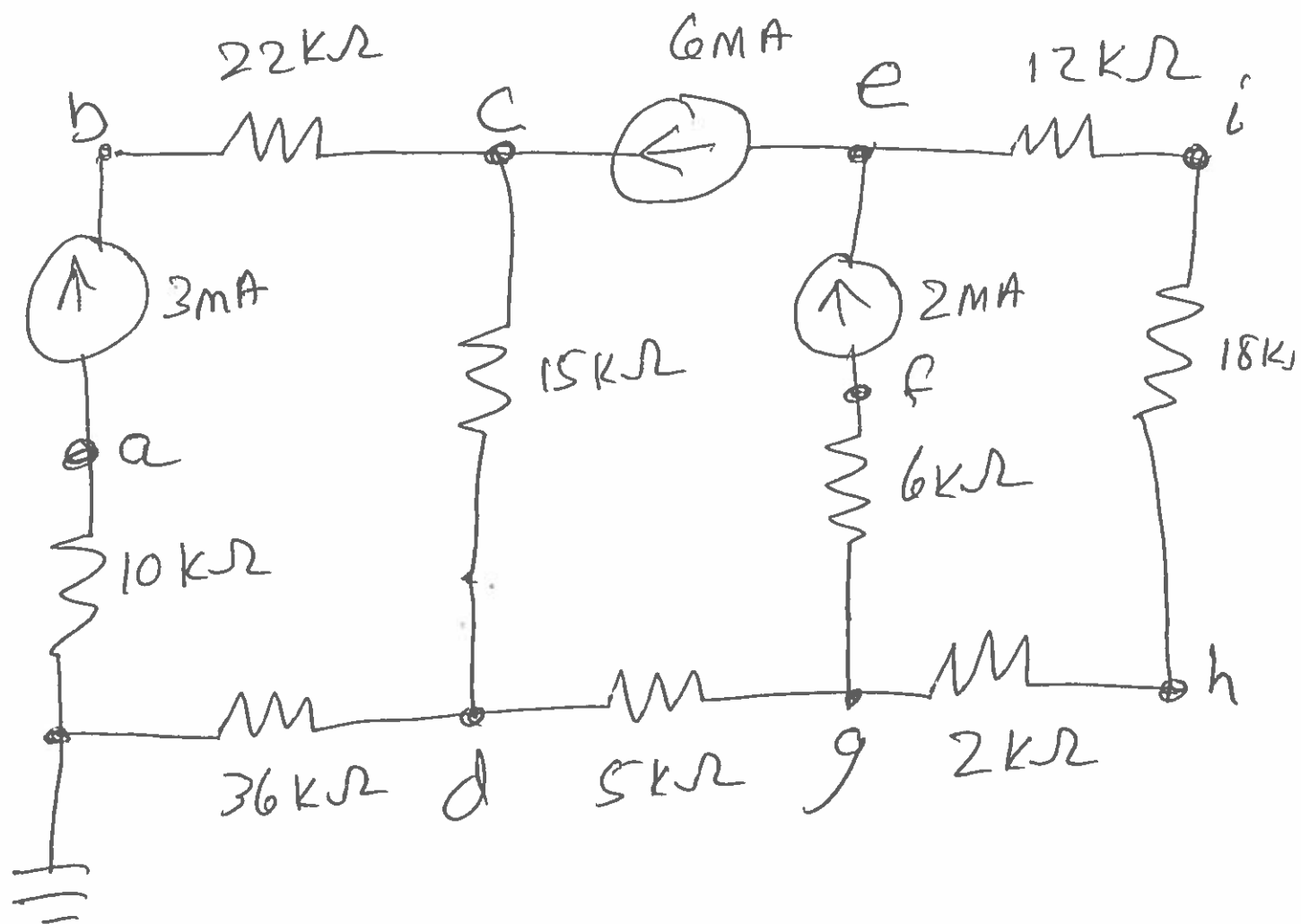
$$i_2 = (15V) \left( \frac{1}{15\Omega} + \frac{1}{5\Omega} \right) - (18V) \left( \frac{1}{15\Omega} \right) - 4V \left( \frac{1}{5\Omega} \right)$$

$$= 15V \left( \frac{4}{15\Omega} \right) - \frac{18V}{15\Omega} - \frac{4V}{5\Omega}$$
$$\frac{12V}{15\Omega}$$

~~Multiply by~~

$$i_2 = \frac{(60 - 18 - 12)V}{15\Omega} = \frac{30V}{15\Omega}$$

$$\boxed{i_2 = +2A}$$



$$a/ -3\text{mA} = V_a \left( \frac{1}{10\text{k}\Omega} \right)$$

$$b/ +3\text{mA} = V_b \left( \frac{1}{22\text{k}\Omega} \right) - V_c \left( \frac{1}{22\text{k}\Omega} \right)$$

$$c/ +6\text{mA} = V_c \left( \frac{1}{22\text{k}\Omega} + \frac{1}{15\text{k}\Omega} \right) - V_b \left( \frac{1}{22\text{k}\Omega} \right) - V_d \left( \frac{1}{15\text{k}\Omega} \right)$$

$$d/ 0 = V_d \left( \frac{1}{5\text{k}\Omega} + \frac{1}{15\text{k}\Omega} + \frac{1}{36\text{k}\Omega} \right) - V_g \left( \frac{1}{5\text{k}\Omega} \right) - V_c \left( \frac{1}{15\text{k}\Omega} \right)$$

$$e / +2\text{mA} - 6\text{mA} = v_d \left( \frac{1}{12\text{k}\Omega} \right) - v_i \left( \frac{1}{12\text{k}\Omega} \right)$$

$$f / -2\text{mA} = v_f \left( \frac{1}{6\text{k}\Omega} \right) - v_g \left( \frac{1}{6\text{k}\Omega} \right)$$

$$g / 0 = v_g \left( \frac{1}{6\text{k}\Omega} + \frac{1}{5\text{k}\Omega} + \frac{1}{2\text{k}\Omega} \right) - v_f \left( \frac{1}{6\text{k}\Omega} \right) \\ - v_d \left( \frac{1}{5\text{k}\Omega} \right) - v_h \left( \frac{1}{2\text{k}\Omega} \right)$$