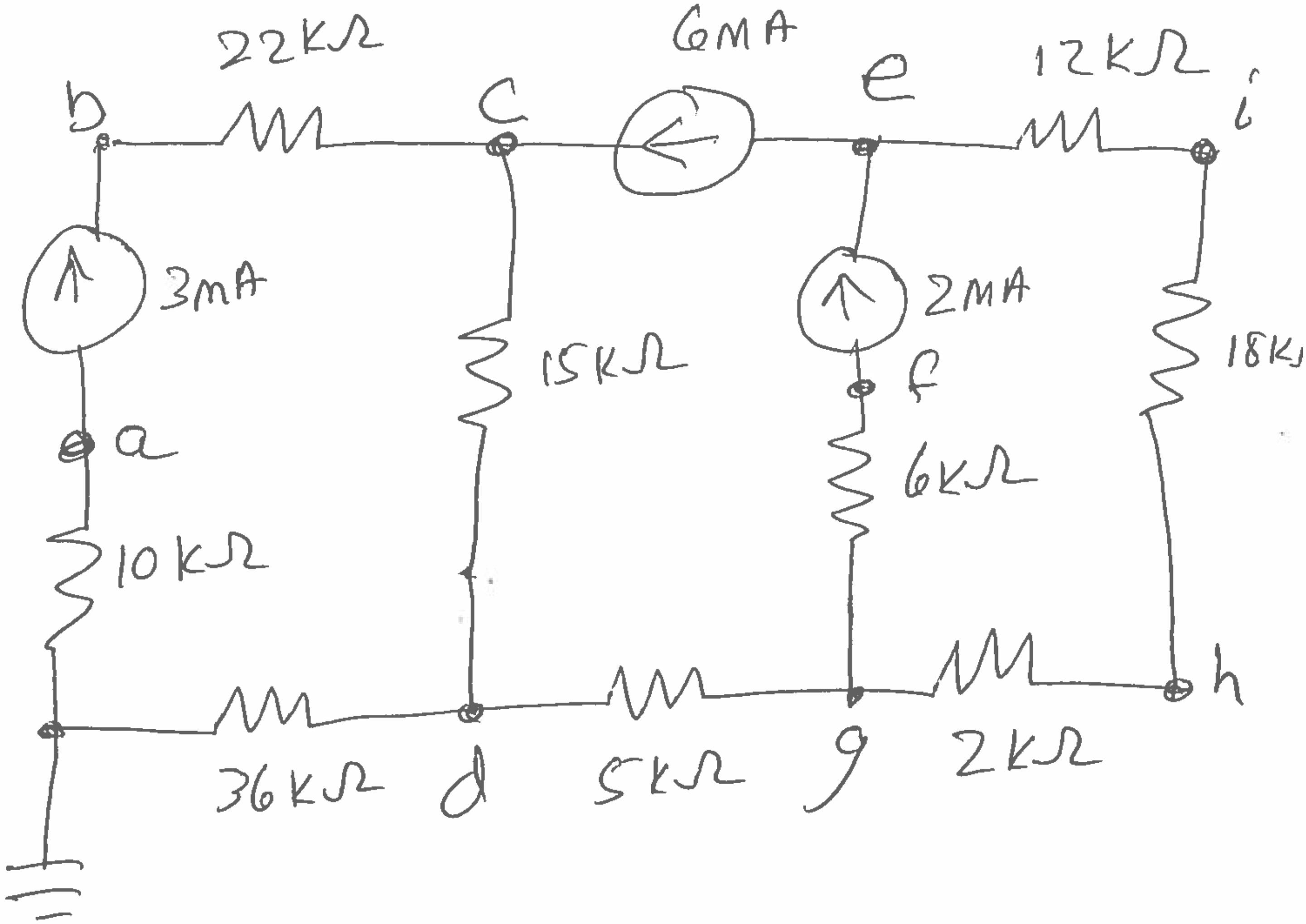


a/ $-3mA = V_a\left(\frac{1}{10k\Omega}\right)$

b/ $+3mA = V_b\left(\frac{1}{22k\Omega}\right) - V_c\left(\frac{1}{22k\Omega}\right)$

c/ $+6mA = V_c\left(\frac{1}{22k\Omega} + \frac{1}{15k\Omega}\right) - V_b\left(1/22k\Omega\right)$
$- V_d\left(1/15k\Omega\right)$

d/ $0 = V_d\left(\frac{1}{5k\Omega} + \frac{1}{15k\Omega} + \frac{1}{36k\Omega}\right) - V_g\left(1/5k\Omega\right)$
$- V_c\left(1/15k\Omega\right)$

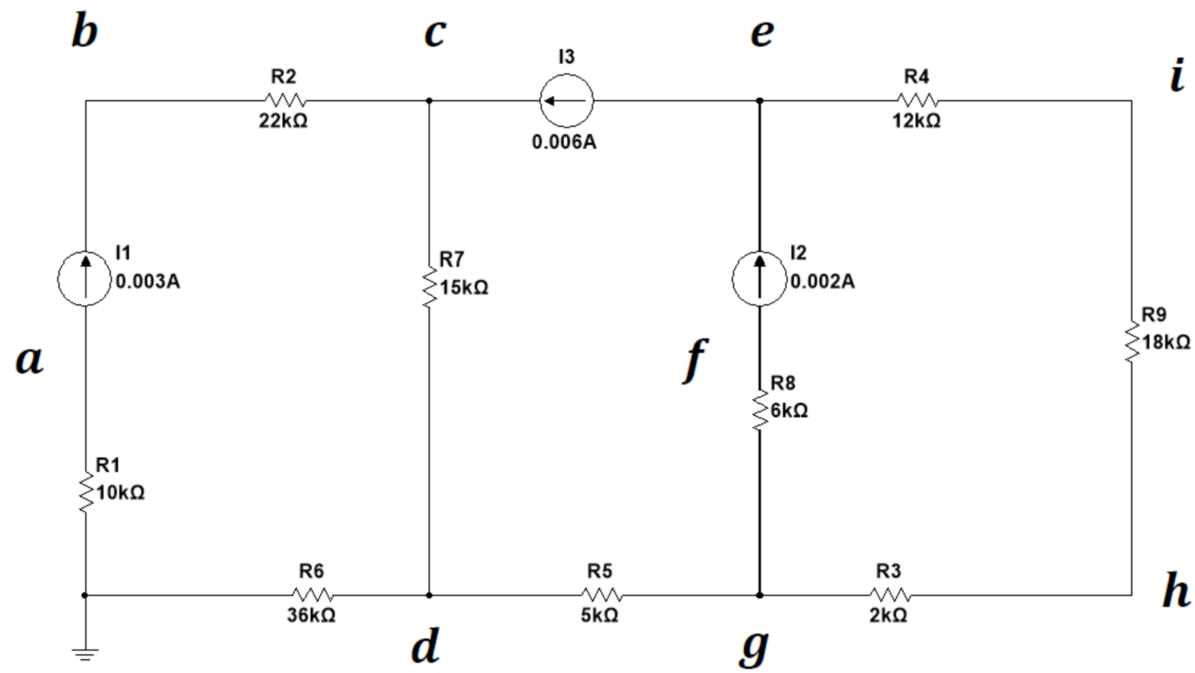
e/ $+2mA - 6mA = v_e\left(\frac{1}{12k\Omega}\right) - v_i\left(\frac{1}{12k\Omega}\right)$

f/ $-2mA = v_f\left(\frac{1}{6k\Omega}\right) - v_g\left(1/6k\Omega\right)$

g/ $0 = v_g\left(\frac{1}{6k\Omega} + \frac{1}{5k\Omega} + \frac{1}{2k\Omega}\right) - v_f\left(1/6k\Omega\right)$

$- v_d\left(\frac{1}{5k\Omega}\right) - v_h\left(1/2k\Omega\right)$

Voltage Node Example #4



Voltage Node Equations

At node a:

$$-3\ mA = V_a\left(\frac{1}{10\ k\Omega}\right)$$

At node b:

$$+3\ mA = V_b\left(\frac{1}{22\ k\Omega}\right) - V_c\left(\frac{1}{22\ k\Omega}\right)$$

At node c:

$$+6\ mA = V_c\left(\frac{1}{22\ k\Omega} + \frac{1}{15\ k\Omega}\right) - V_b\left(\frac{1}{22\ k\Omega}\right) - V_d\left(\frac{1}{15\ k\Omega}\right)$$

At node d:

$$0 = V_d\left(\frac{1}{5\ k\Omega} + \frac{1}{15\ k\Omega} + \frac{1}{36\ k\Omega}\right) - V_g\left(\frac{1}{5\ k\Omega}\right) - V_c\left(\frac{1}{15\ k\Omega}\right)$$

At node e:

$$+2\ mA - 6\ mA = V_e\left(\frac{1}{12\ k\Omega}\right) - V_i\left(\frac{1}{12\ k\Omega}\right)$$

At node f:

$$-2\ mA = V_f\left(\frac{1}{6\ k\Omega}\right) - V_g\left(\frac{1}{6\ k\Omega}\right)$$

At node g:

$$0 = V_g\left(\frac{1}{6\ k\Omega} + \frac{1}{5\ k\Omega} + \frac{1}{2\ k\Omega}\right) - V_f\left(\frac{1}{6\ k\Omega}\right) - V_d\left(\frac{1}{5\ k\Omega}\right) - V_h\left(\frac{1}{2\ k\Omega}\right)$$

At node h:

$$0 = V_h\left(\frac{1}{2\ k\Omega} + \frac{1}{18\ k\Omega}\right) - V_g\left(\frac{1}{2\ k\Omega}\right) - V_i\left(\frac{1}{18\ k\Omega}\right)$$

At node i:

$$0 = V_i\left(\frac{1}{12\ k\Omega} + \frac{1}{18\ k\Omega}\right) - V_e\left(\frac{1}{12\ k\Omega}\right) - V_h\left(\frac{1}{18\ k\Omega}\right)$$

Rewriting to make entering into MATLAB easier:

$$-3\ mA = V_a\left(\frac{1}{10\ k\Omega}\right)$$

$$+3\ mA = V_b\left(\frac{1}{22\ k\Omega}\right) - V_c\left(\frac{1}{22\ k\Omega}\right)$$

$$+6\ mA = -V_b\left(\frac{1}{22\ k\Omega}\right) + V_c\left(\frac{1}{22\ k\Omega} + \frac{1}{15\ k\Omega}\right) - V_d\left(\frac{1}{15\ k\Omega}\right)$$

$$0 = -V_c\left(\frac{1}{15\ k\Omega}\right) + V_d\left(\frac{1}{5\ k\Omega} + \frac{1}{15\ k\Omega} + \frac{1}{36\ k\Omega}\right) - V_g\left(\frac{1}{5\ k\Omega}\right)$$

$$+2\ mA - 6\ mA = V_e\left(\frac{1}{12\ k\Omega}\right) - V_i\left(\frac{1}{12\ k\Omega}\right)$$

$$-2\ mA = V_f\left(\frac{1}{6\ k\Omega}\right) - V_g\left(\frac{1}{6\ k\Omega}\right)$$

$$0 = -V_d\left(\frac{1}{5\ k\Omega}\right) - V_f\left(\frac{1}{6\ k\Omega}\right) + V_g\left(\frac{1}{6\ k\Omega} + \frac{1}{5\ k\Omega} + \frac{1}{2\ k\Omega}\right) - V_h\left(\frac{1}{2\ k\Omega}\right)$$

$$0 = -V_g\left(\frac{1}{2\ k\Omega}\right) + V_h\left(\frac{1}{2\ k\Omega} + \frac{1}{18\ k\Omega}\right) - V_i\left(\frac{1}{18\ k\Omega}\right)$$

$$0 = -V_e\left(\frac{1}{12\ k\Omega}\right) - V_h\left(\frac{1}{18\ k\Omega}\right) + V_i\left(\frac{1}{12\ k\Omega} + \frac{1}{18\ k\Omega}\right)$$

From MATLAB the Voltage nodes are found to be:

Node	Voltage (V)
a	-30.
b	309.
c	243.
d	108
e	-50.
f	66.
g	78.
h	70.
i	-2.0

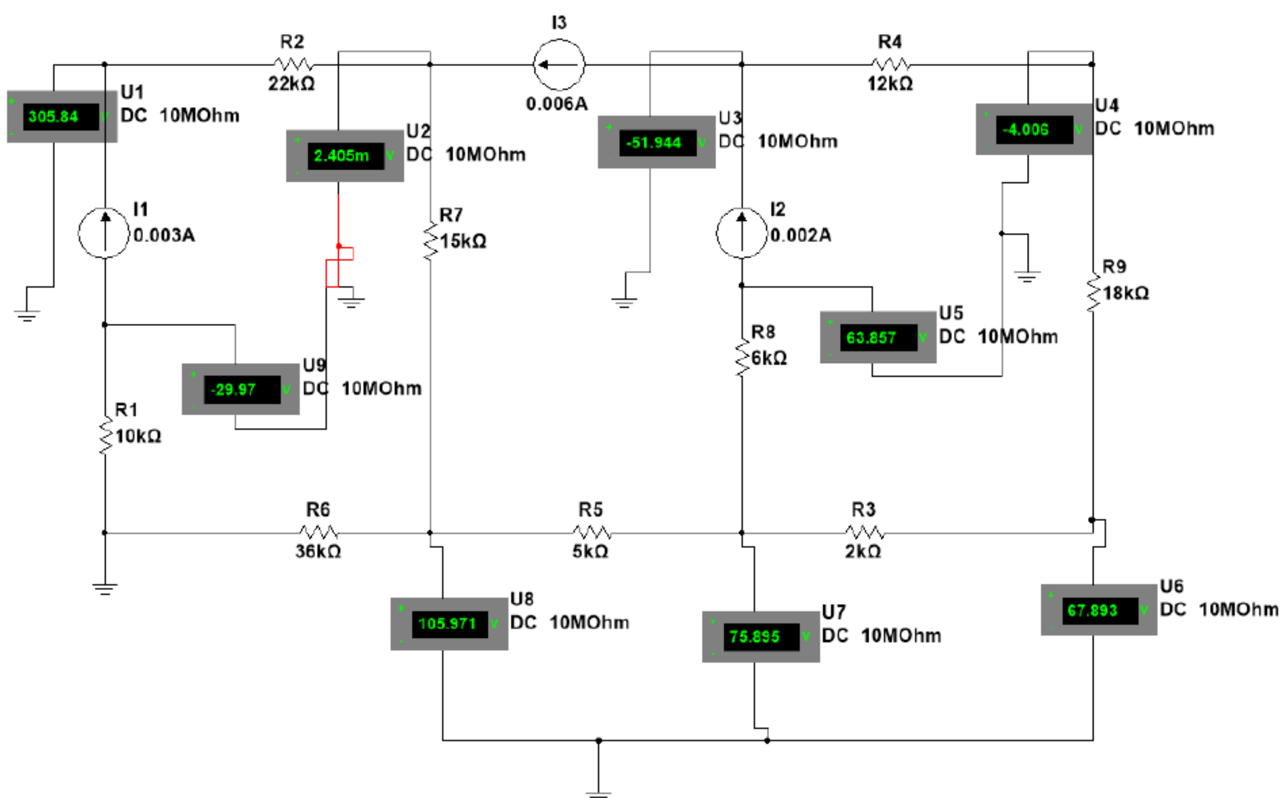
MATLAB Code

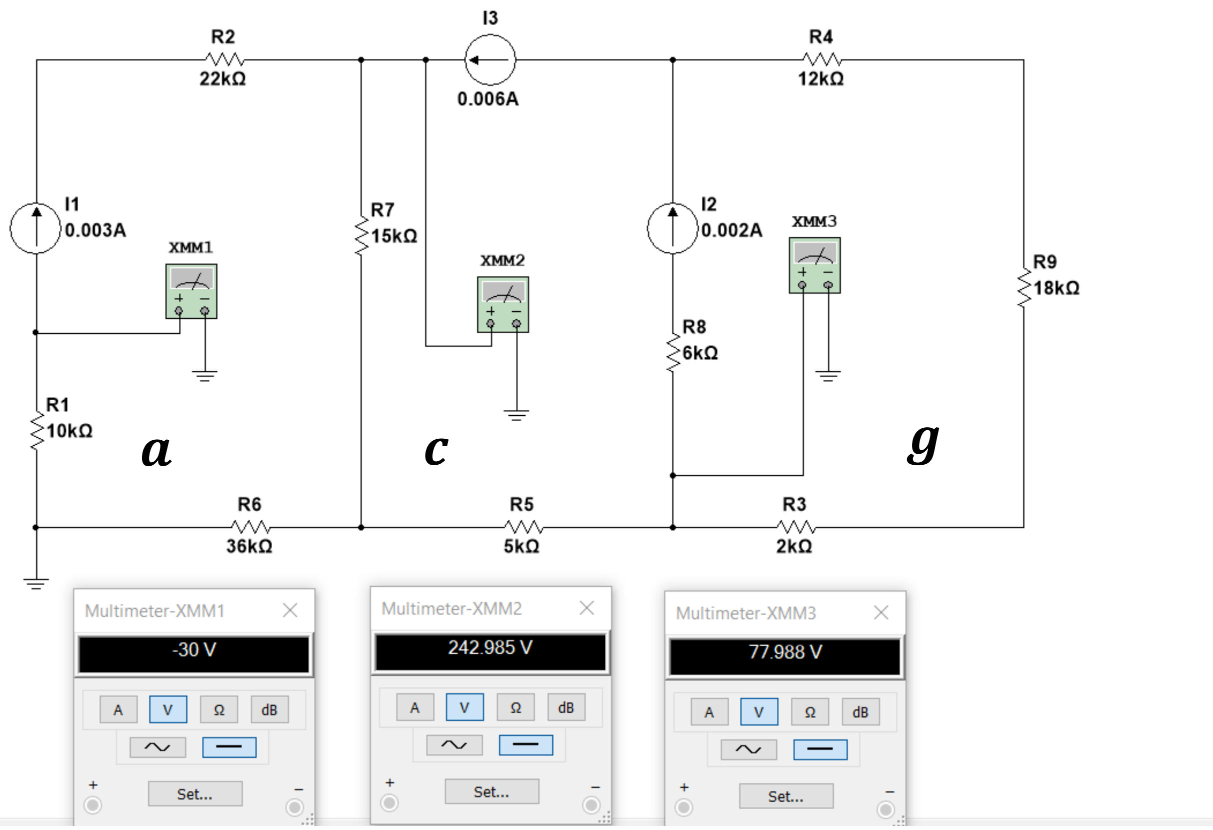
```
%lecture example of Voltage nodes with constant Current sources
%large circuit.  Example 4
%version 2021-02-12 DW Donovan
```

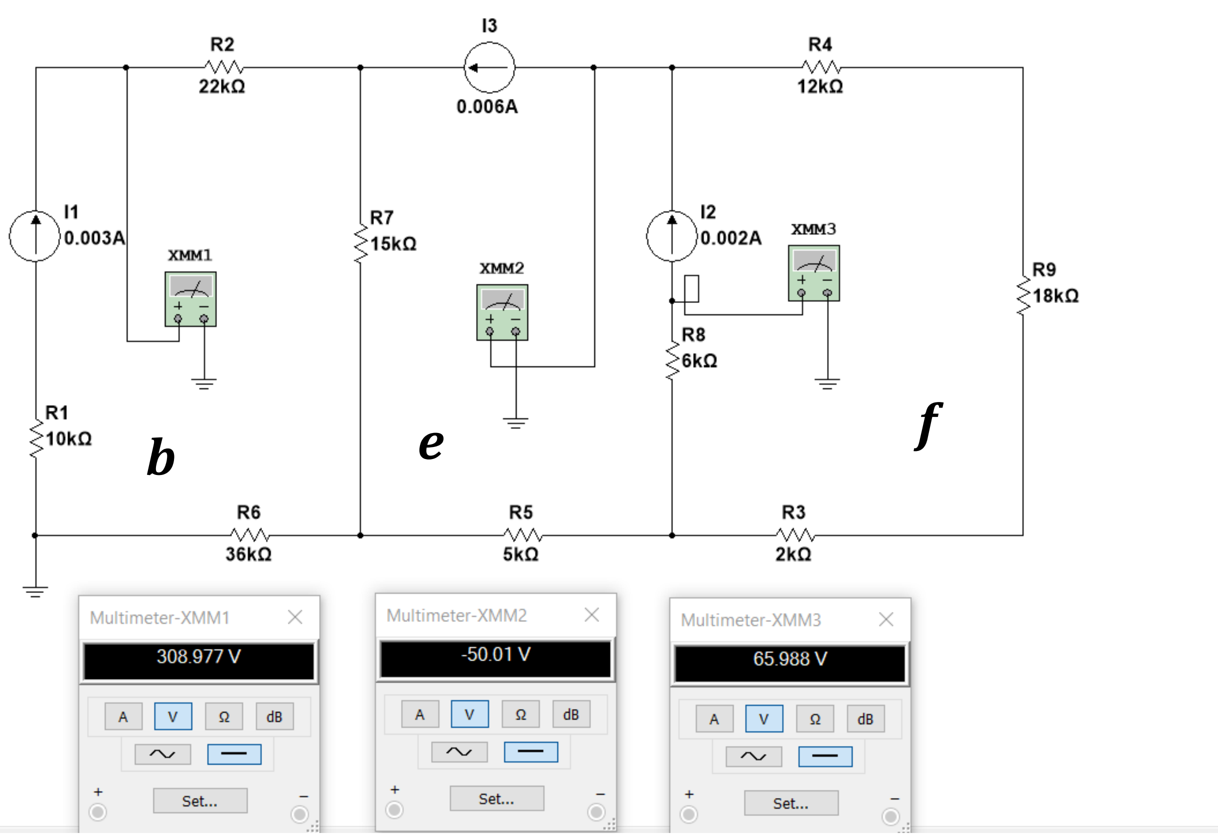
```
clear all;

%Voltage Node Method
G=[(1/10) 0 0 0 0 0 0 0 0;
    0 (1/22) (-1/22) 0 0 0 0 0 0;
    0 (-1/22) (1/22 + 1/15) (-1/15) 0 0 0 0 0;
    0 0 (-1/15) (1/5 + 1/15 + 1/36) 0 0 (-1/5) 0 0;
    0 0 0 0 (1/12) 0 0 0 (-1/12);
    0 0 0 0 0 (1/6) (-1/6) 0 0;
    0 0 0 (-1/5) 0 (-1/6) (1/5 + 1/6 + 1/2) (-1/2) 0;
    0 0 0 0 0 0 (-1/2) (1/2 + 1/18) (-1/18);
    0 0 0 0 (-1/12) 0 0  (-1/18) (1/12 + 1/18)];
ii=[-3 3 6 0 (2-6) -2 0 0 0]';
V=G\ii;
va=V(1);
vb=V(2);
vc=V(3);
vd=V(4);
ve=V(5);
vf=V(6);
vg=V(7);
vh=V(8);
vi=V(9);

ANS={'Node' 'Voltage (V)';
    'va' va;
    'vb' vb;
    'vc' vc;
    'vd' vd;
    've' ve;
    'vf' vf;
    'vg' vg;
    'vh' vh;
    'vi' vi};
ANS
```

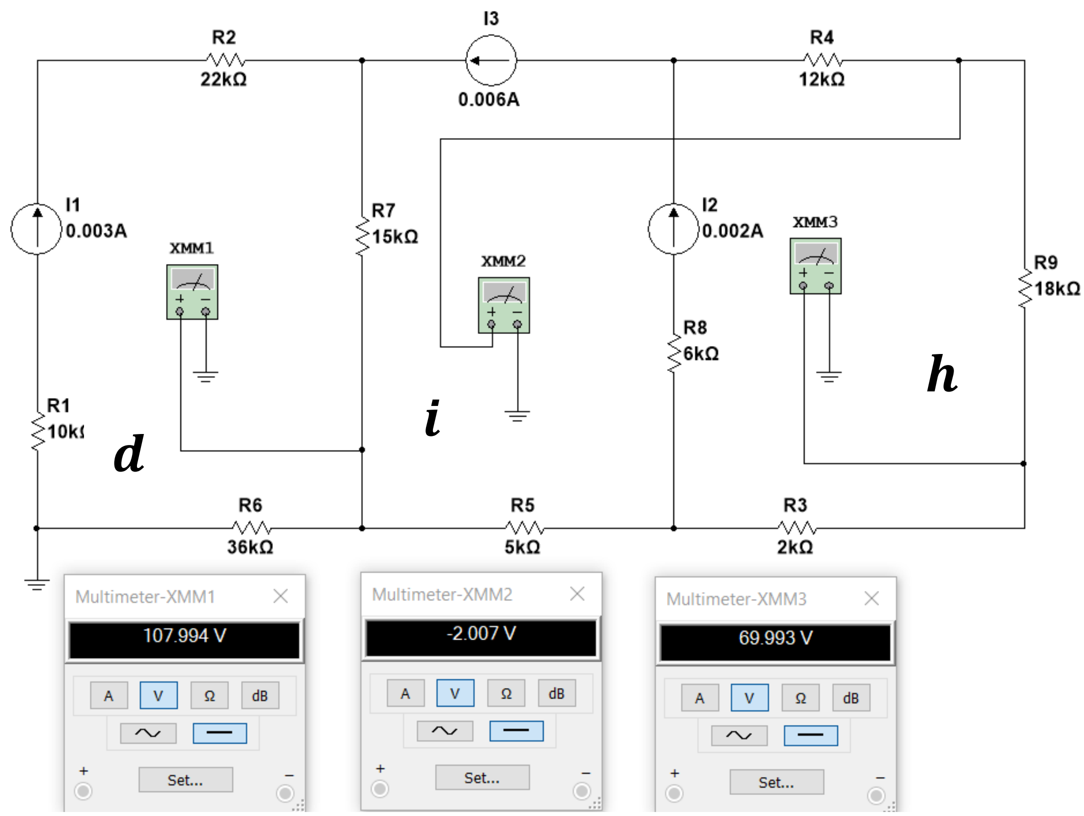
```
%{
ANS =
    'Node'     'Voltage (V)'
    'va'       [        -30]
    'vb'       [   309.0000]
    'vc'       [   243.0000]
    'vd'       [   108.0000]
    've'       [   -50.0000]
    'vf'       [    66.0000]
    'vg'       [    78.0000]
    'vh'       [    70.0000]
    'vi'       [    -2.0000]
%}

%Double Check

i1 = 3;
i2 = -6;
i3 = -4;
vac = -i1*10;
vbc = +i1*36+(i1-i2)*15+i1*22;
vcc = +i1*36+(i1-i2)*15;
vdc = +i1*36;
vec = +i1*36+i2*5+i3*2+i3*18+i3*12;
vfc = +i1*36+i2*5+(i2-i3)*6;
vgc = +i1*36+i2*5;
vhc = +i1*36+i2*5+i3*2;
vic = +i1*36+i2*5+i3*2+i3*18;

Ans2 = {'Node' 'Voltage (V)' 'Check Voltage (V)';
    'va' va  vac;
    'vb' vb  vbc;
    'vc' vc  vcc;
    'vd' vd vdc;
    've' ve vec;
    'vf' vf vfc;
    'vg' vg vgc;
    'vh' vh vhc;
    'vi' vi vic};

Ans2
```

```
%{
Ans2 =
    'Node'     'Voltage (V)'     'Check Voltage (V)'
    'va'       [          -30]     [                -30]
    'vb'       [     309.0000]     [                309]
    'vc'       [     243.0000]     [                243]
    'vd'       [     108.0000]     [                108]
    've'       [     -50.0000]     [                -50]
    'vf'       [      66.0000]     [                 66]
    'vg'       [      78.0000]     [                 78]
    'vh'       [      70.0000]     [                 70]
    'vi'       [      -2.0000]     [                 -2]

%}
```




R2
22kΩ
I3
0.006A
R4
12kΩ
I1
0.003A
XMM1
R7
15kΩ
XMM2
I2
0.002A
XMM3
R9
18kΩ
R8
6kΩ
R1
10kΩ
a
c
g
R6
36kΩ
R5
5kΩ
R3
2kΩ
Multimeter-XMM1
-30 V
Multimeter-XMM2
242.985 V
Multimeter-XMM3
77.988 V
A V Ω dB
Set...

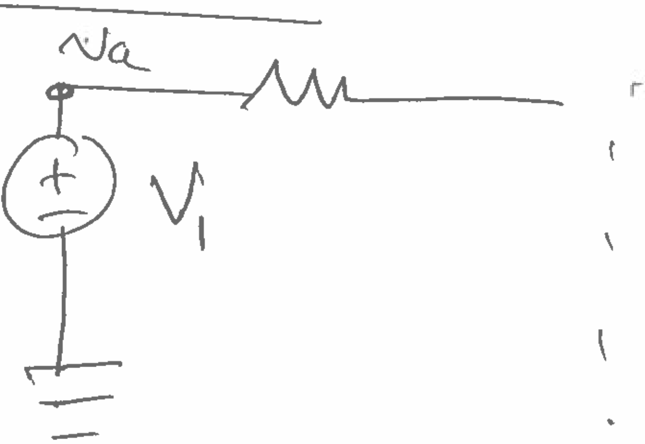

R2
22kΩ
I3
0.006A
R4
12kΩ
I1
0.003A
XMM1
R7
15kΩ
XMM2
I2
0.002A
XMM3
R9
18kΩ
R8
6kΩ
R1
10kΩ
b
e
f
R6
36kΩ
R5
5kΩ
R3
2kΩ
Multimeter-XMM1
308.977 V
Multimeter-XMM2
-50.01 V
Multimeter-XMM3
65.988 V
A V Ω dB
Set...


I3
R2
22kΩ
0.006A
R4
12kΩ
I1
0.003A
XMM1
R7
15kΩ
XMM2
I2
0.002A
XMM3
R9
18kΩ
R8
6kΩ
h
R1
10k
d
i
R6
36kΩ
R5
5kΩ
R3
2kΩ
Multimeter-XMM1
107.994 V
A
V
Ω
dB
Set...
Multimeter-XMM2
-2.007 V
A
V
Ω
dB
Set...
Multimeter-XMM3
69.993 V
A
V
Ω
dB
Set...

Voltage node when voltage sources present.

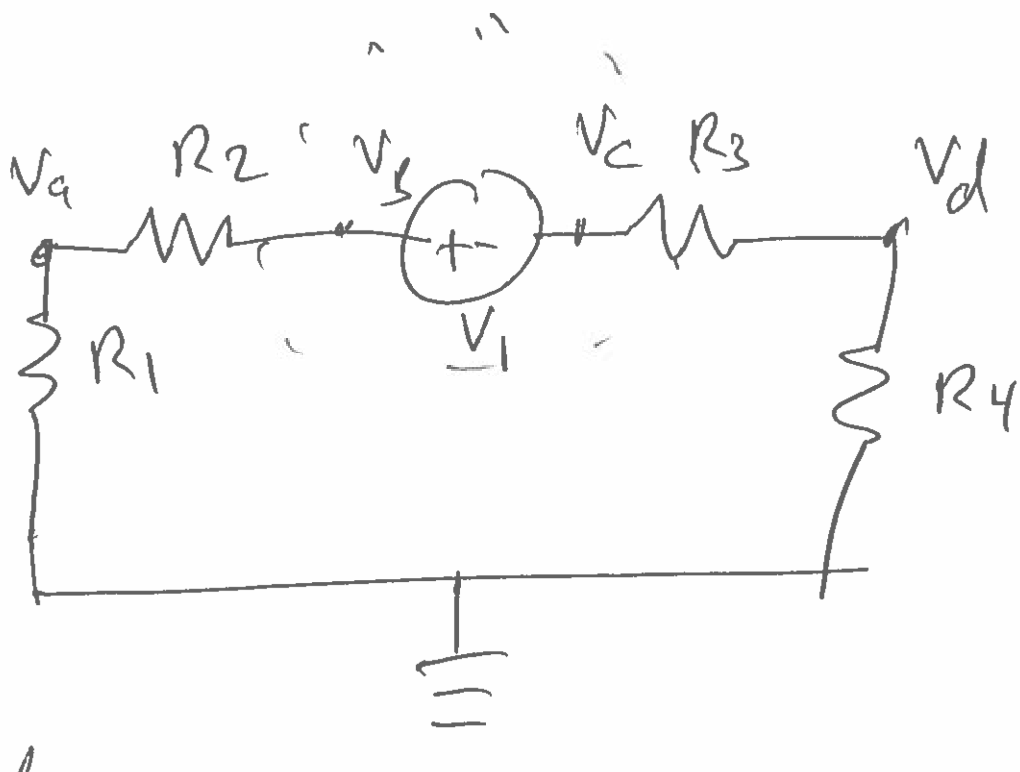
Two possibilities

(1)



$v_a = V_1$

(2)



Supernode B-C

2 equations

① $\rightarrow V_B = V_C + V_1$

② Treat both V_B and V_C as Positive

$$\rightarrow 0 = V_B(1/R_2) + V_C\left(\frac{1}{R_3}\right) - V_A(1/R_2)$$

$$- V_d\left(\frac{1}{R_3}\right)$$

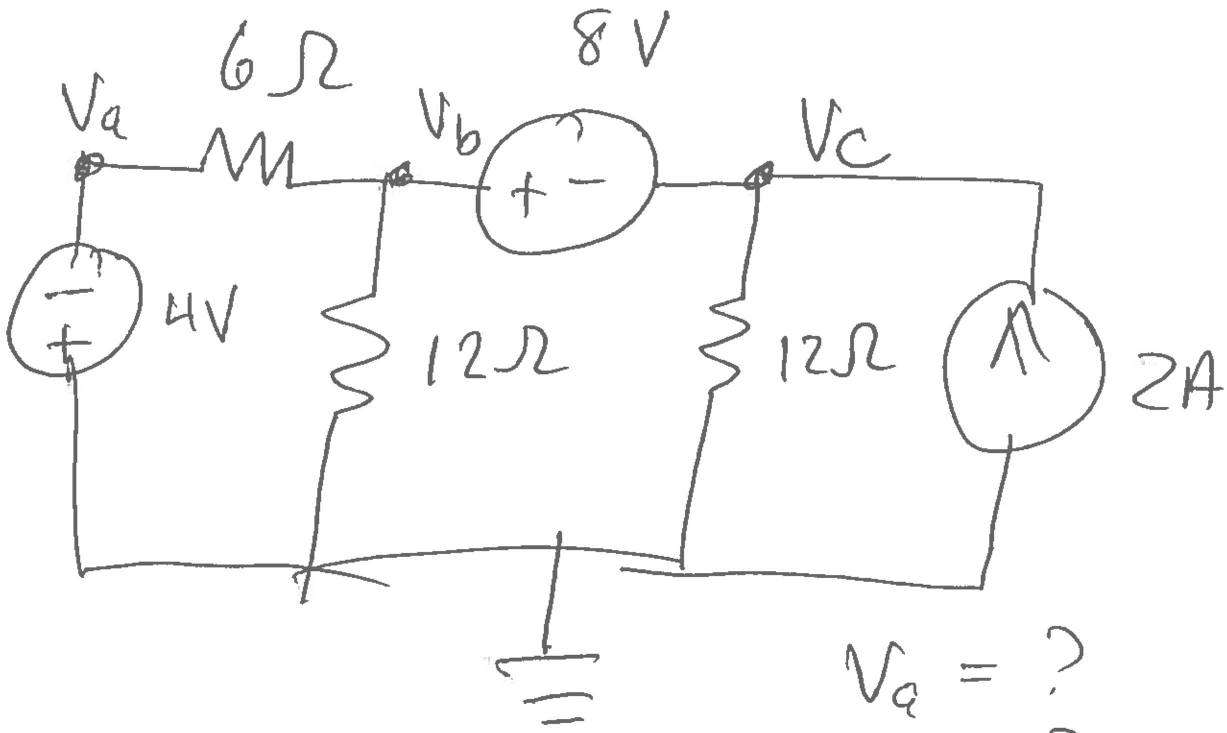
rest

$$\cancel{0 = V_A\left(\frac{1}{R_2}\right) - V_B(1/R_2)}$$

$$\cancel{0 = V_D(1/R_3) - V_C(1/R_3)}$$

$$\rightarrow 0 = V_A\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_B(1/R_2)$$

$$\rightarrow 0 = V_D\left(\frac{1}{R_3} + \frac{1}{R_4}\right) - V_C(1/R_3)$$



$V_a = ?$

$V_b = ?$

$V_c = ?$

$$\boxed{V_a = -4V}$$

Supernode $\Rightarrow$ $V_b = V_c + 8\,V$

$$2\,A = V_b\left(\frac{1}{6\Omega} + \frac{1}{12\Omega}\right) + V_c\left(1/12\Omega\right) - V_a\left(1/6\Omega\right)$$

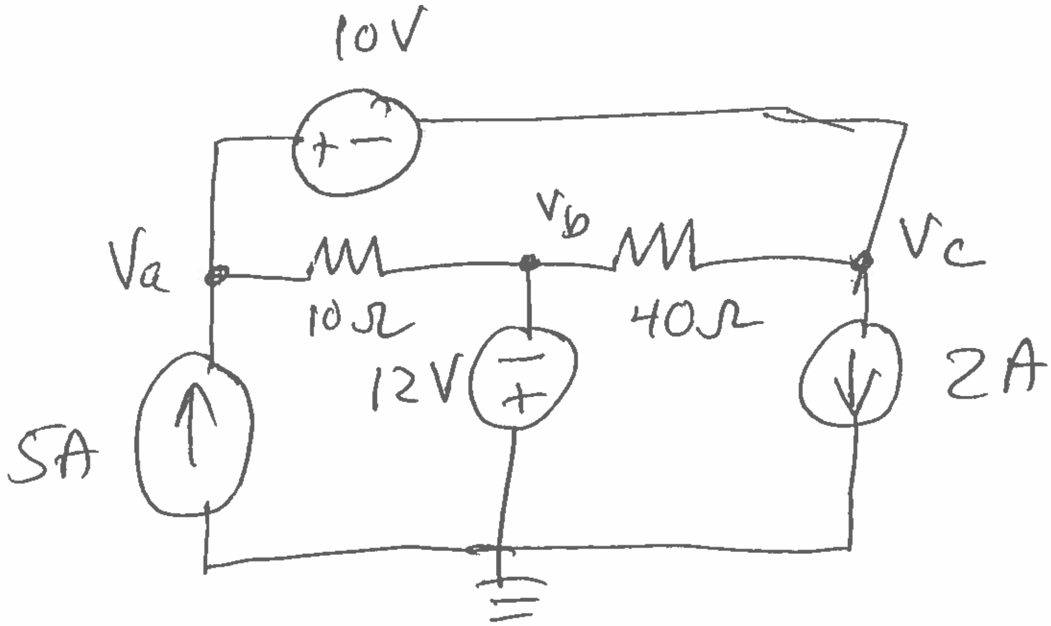
$$2A = \left(V_c + 8V\right)\left(\frac{2+1}{12\Omega}\right) + V_c\left(\frac{1}{12\Omega}\right) - (-4V)\,\frac{2}{12\Omega}$$

$$\cancel{24V} - 8V = 3V_C + \cancel{24V} + V_C$$

$$-8V = 4V_C \Rightarrow \boxed{V_C = -2V}$$

$$V_b = V_C + 8V = -2V + 8V = 6V$$

$$\boxed{V_A = -4V,\ V_B = +6V,\ V_C = -2V}$$



$$V_b = -12V$$

Supernode $\quad V_a = V_c + 10V$

$$5A - 2A = V_A\left(\frac{1}{10\Omega}\right) + V_c\left(\frac{1}{40\Omega}\right) - V_B\left(\frac{1}{10\Omega} + \frac{1}{40\Omega}\right)$$

$$3A = (V_c + 10V)\left(\frac{1}{10\Omega}\right) + V_c\left(\frac{1}{40\Omega}\right) - (-12V)\left(\frac{4+1}{40\Omega}\right)$$

$$120V = 4V_c + 40V + V_c + 60V$$

$$20V = 5V_c \Rightarrow V_c = 4V$$

$$V_a = V_c + 10V = 4V + 10V$$

$$V_a = 14V$$

$$V_b = -12V$$

$$V_c = 4V$$

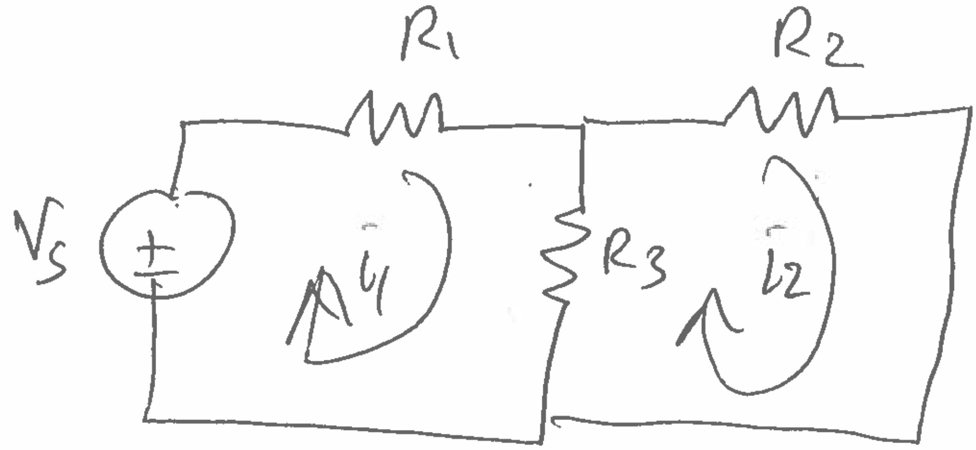
Mesh Currents

- must be planar circuits.

A mesh is a loop which does not contain any other loops with in it.

Convention is all mesh currents go clockwise.

This is primarily another way of using Kirchhoff's loop rule.



Left loop $+V_s - i_1 R_1 - (i_1 - i_2) R_3 = 0$

Right loop $\cancel{(i_2 - i_1)}$ $(i_1 - i_2) R_3 - i_2 R_2 = 0$

$$V_S = i_1(R_1 + R_3) - i_2 R_3$$

$$0 = -i_1(R_3) + i_2(R_3 + R_2)$$