

ideally  $\Rightarrow V_o = G_o (V_{i1} - V_{i2})$

$V_{di} = V_{i1} - V_{i2} \Rightarrow$  differential input signal

$G_o$  is the differential gain

$$G_o = \frac{V_o}{V_{di}}$$

However there also exists  $V_{cmi}$

known as common mode ~~input~~ input voltage

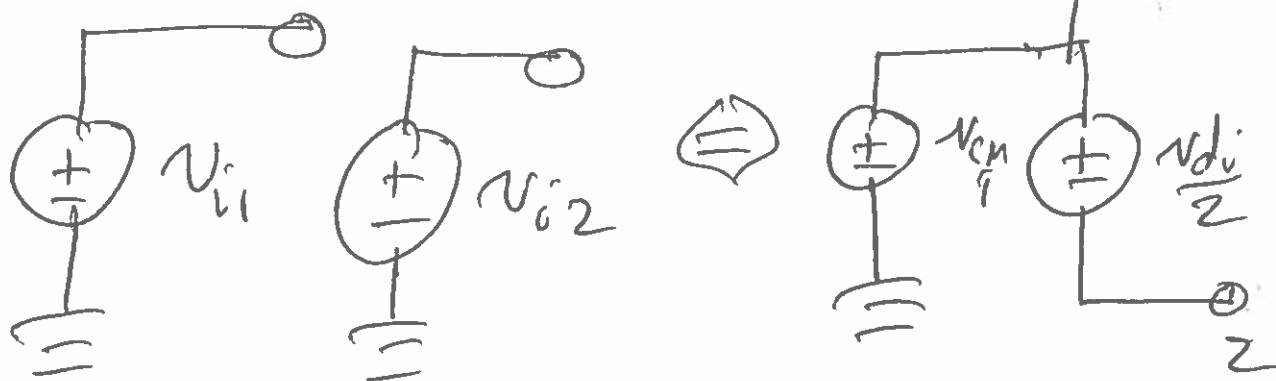
$$V_{cmi} = \frac{V_{i1} + V_{i2}}{2} = \text{average value of } V_{i1}, V_{i2}$$

$$G_{cm} = \frac{V_o}{V_{cmi}}$$

You would prefer  
Amplifier did not  
Amplify this

$$G_{cm} \Rightarrow 0$$

We can Assume



Theoretically  $V_o = G_D V_{Di} + G_{cm} V_{cmi}$

We like  $G_{cm} \Rightarrow 0$

CMRR - Common mode Rejection Ratio

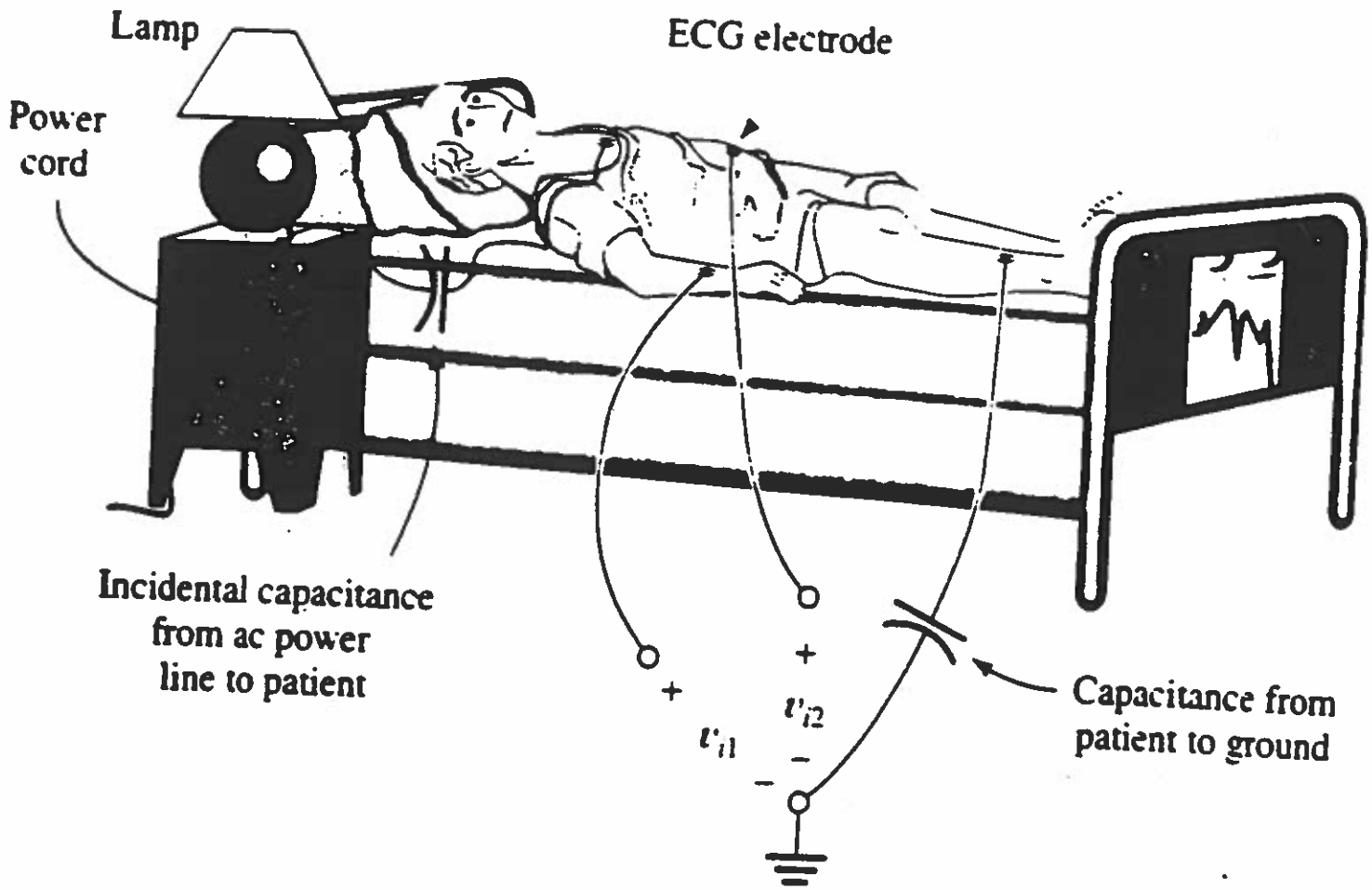
$$CMRR = 20 \log \left( \frac{|G_D|}{|G_{cm}|} \right) = f(\omega) \uparrow \text{freq}$$

At 60Hz CMRR of  $\sim 120$  dB is considered good.

$$120 \text{ dB} = 20 \log \left( \frac{G_D}{G_{CM}} \right)$$

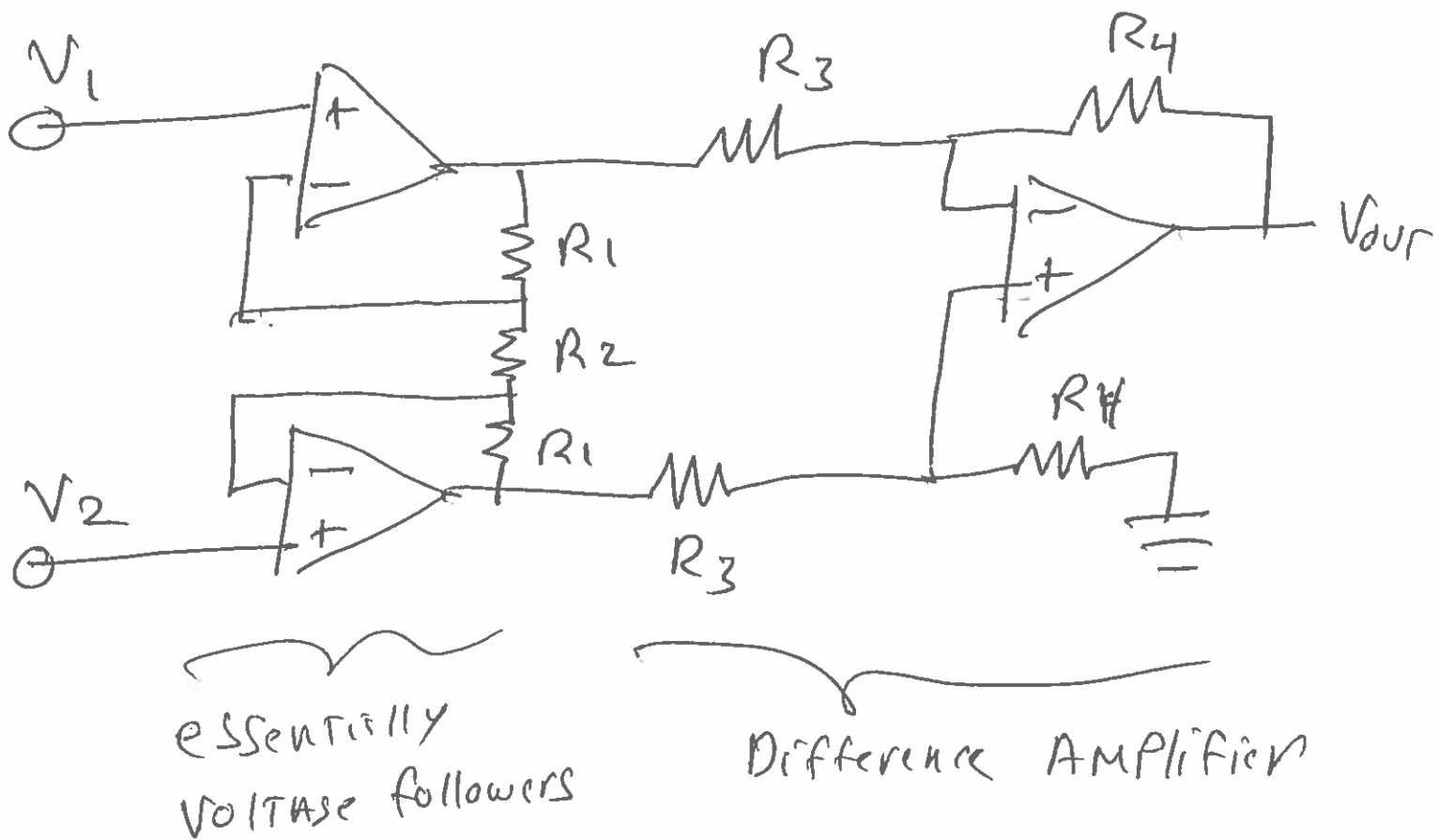
$$6 = \log \left( \frac{G_D}{G_{CM}} \right)$$

$$10^6 = \frac{G_D}{G_{CM}} \Rightarrow G_D = 10^6 G_{CM}$$



**Figure 1.45** Electrocardiographs encounter large 60-Hz common-mode signals.

To fix this situation we use an  
INSTRUMENTATION AMPLIFIER,



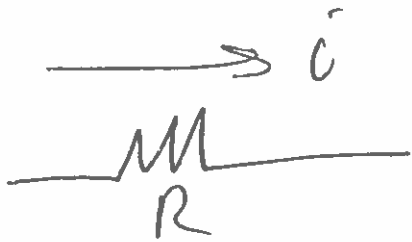
$$V_{OUT} = \frac{R_4}{R_3} \left( 1 + 2 \frac{R_1}{R_2} \right) (V_2 - V_1)$$

if  $R_2 = 2R_1$

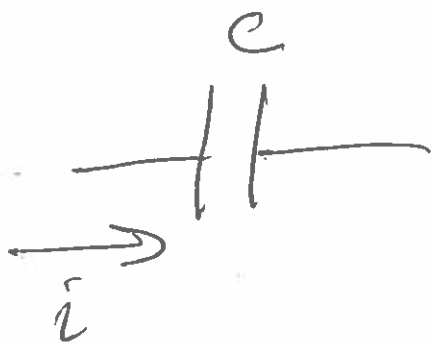
$$V_{OUT} = 2 \left( \frac{R_4}{R_3} \right) (V_2 - V_1)$$

Difference AMP has relatively low impedance  
 INSTRUMENTATION AMP has high impedance

Resistor matching much less critical in  
INSTRUMENTATION AMP Than in Difference AMP.



Ohm's Law  $\Rightarrow i = \frac{1}{R} V$



in D.C. current flows  
until capacitor is  
fully charged.

in A.C. current flows  
during periods when capacitor  
is not fully charged

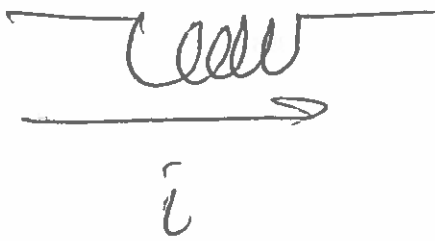
$i = \frac{1}{X_c} V$

Ohm's Law now

$X_c$  is called capacitive reactance.

$X_c = \frac{V}{i} = \frac{1}{\omega C}$

so as  $\omega \uparrow$   $X_c \downarrow$   
as  $\omega \rightarrow 0$   $X_c \rightarrow \infty$   
open circuit



When  $i$  starts  $\hat{v}_{induced}$  opposes  $i$

$$i = \frac{1}{X_L} V$$

AT DC  $i$  is constant  
coil appears to be  
wire (short circuit)

$$X_L = i \omega L$$

as  $\omega \rightarrow \infty$   $X_L \rightarrow \infty$   
open circuit

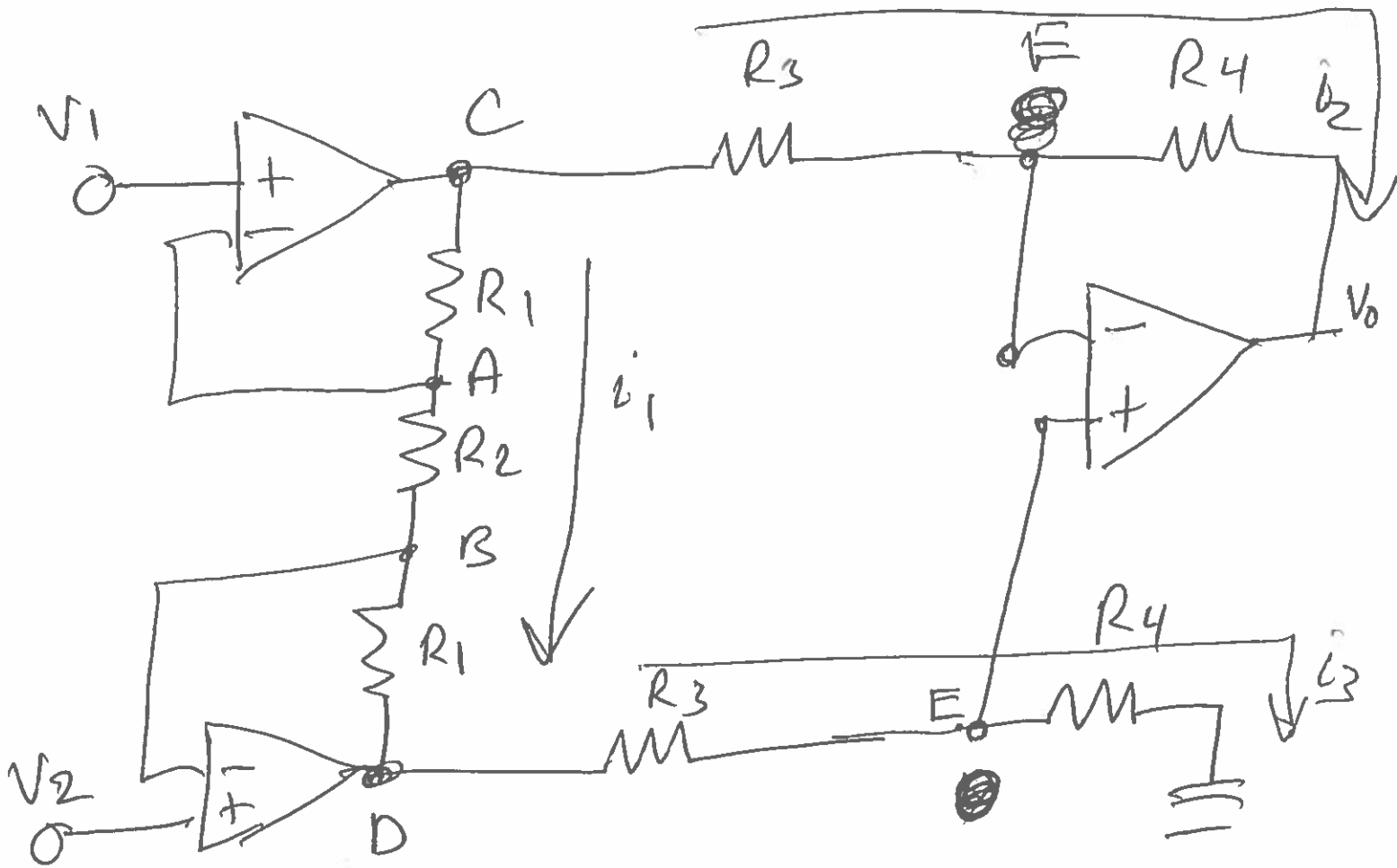
as  $\omega \rightarrow 0$   $X_L \rightarrow 0$   
short circuit

Impedance  $Z$

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Ohm's Law in AC  $\Rightarrow i = \frac{1}{Z} V$

$\frac{E}{L} i$  The  $i C E$  Men  
 $\begin{matrix} \rightarrow & \uparrow & - \\ & & \end{matrix}$ 
 $\begin{matrix} \rightarrow & \uparrow & - \\ & & \end{matrix}$



$$i_1 = \frac{V_1 - V_2}{R_2}$$

$$V_C = V_1 + i_1 R_1$$

$$V_D = V_2 - i_1 R_1$$

$$V_C = V_1 + \frac{V_1 - V_2}{R_2} R_1 = V_1 \left(1 + \frac{R_1}{R_2}\right) - V_2 \left(\frac{R_1}{R_2}\right)$$

$$V_C = V_1 \left(1 + \frac{R_1}{R_2}\right) - V_2 \left(\frac{R_1}{R_2}\right)$$

$$V_D = V_2 \left(1 + \frac{R_1}{R_2}\right) - V_1 \left(\frac{R_1}{R_2}\right)$$

$$i_3 = \frac{V_D - V_E}{R_3} = \frac{V_E}{R_4}$$

6

$$V_D = V_E \left( \frac{R_3}{R_4} + 1 \right)$$

$$V_E = V_D \left( \frac{R_4}{R_4 + R_3} \right)$$

$$i_2 = \frac{V_C - V_E}{R_3} = \frac{V_E - V_D}{R_4}$$

$$\frac{R_4}{R_3} V_C - \frac{R_4}{R_3} V_E = V_E - V_D$$

$$V_D = V_E \left( 1 + \frac{R_4}{R_3} \right) - V_C \frac{R_4}{R_3}$$

$$V_o = V_D \left( \frac{R_4}{R_4 + R_3} \right) \left( \frac{R_3 + R_4}{R_3} \right) - V_C \frac{R_4}{R_3}$$

$$V_o = \frac{R_4}{R_3} (V_D - V_C)$$

$$V_o = \frac{R_4}{R_3} \left[ \left( V_2 \left( 1 + \frac{R_1}{R_2} \right) - V_1 \left( \frac{R_1}{R_2} \right) \right) \right.$$

$$\left. - \left( V_1 \left( 1 + \frac{R_1}{R_2} \right) - V_2 \left( \frac{R_1}{R_2} \right) \right) \right]$$

$$= \frac{R_4}{R_3} \left[ V_2 \left( 1 + \frac{R_1}{R_2} + \frac{R_1}{R_2} \right) - V_1 \left( \left( 1 + \frac{R_1}{R_2} \right) + \frac{R_1}{R_2} \right) \right]$$

$$V_o = \frac{R_4}{R_3} \left( 1 + \frac{2R_1}{R_2} \right) (V_2 - V_1)$$

# Advantages of INSTRUMENTATION AMP

$$1) G_D = 1 + \frac{R_1}{R_2}$$

$$G_{cm} = 1 \quad \text{if} \quad \frac{R_1}{R_2} \gg 1$$

$$G_D \gg G_{cm}$$