Energy Storage Elements – capacitors, inductors

<u>Capacitors</u> Two conductors separated by a dielectric store charge which provides storing energy in an electric field.

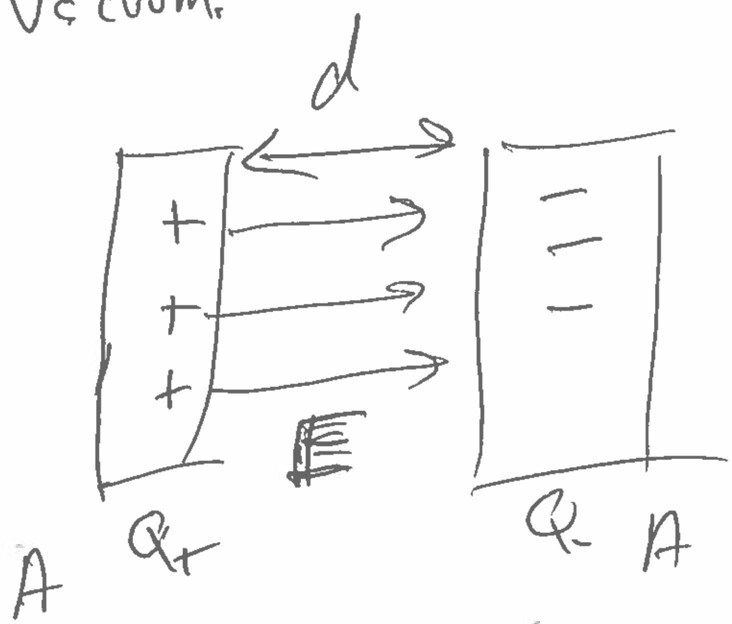
Primary purpose of a capacitor is to maintain the local voltage.

If voltage locally increases capacitor takes more charge onto conductors if voltage locally decreases capacitor releases charge from conductors.

$$Q = CV$$

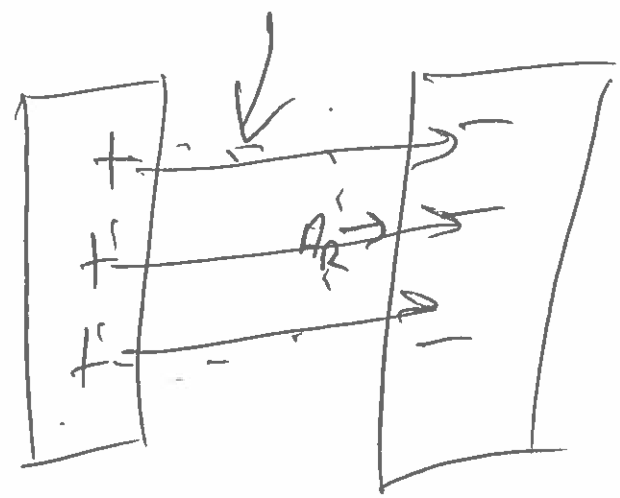
How to find C ?

Consider Parallel Plate Capacitor separated by a vacuum.



$|Q_+| = |Q_-|$

$E = ?$ Use Gauss's Law



$$\Phi_{surface} = E A_R = \frac{Q_{enc}}{\varepsilon_0}$$

Make A_R size of Plate

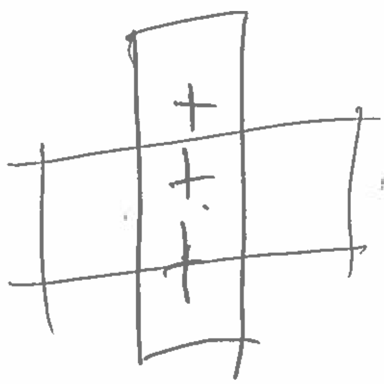
$Q_{enc} = Q$ on Plate

If A not Plate Area

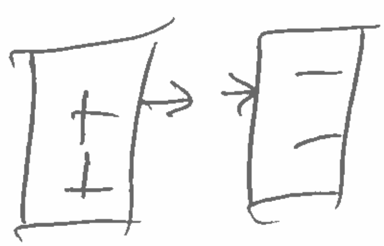
$$Q_{enc} = \sigma A_R$$

$$E A_R = \frac{\sigma A_R}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$ which is field inside capacitor



$$E_{cond} = \frac{\sigma}{2\varepsilon_0}$$



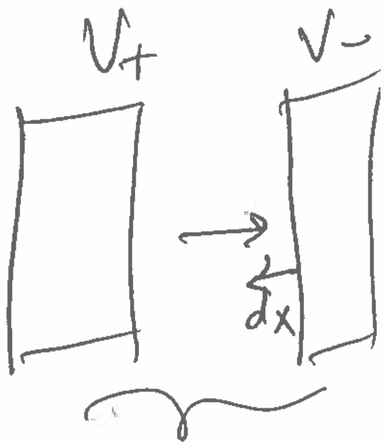
$$\frac{\sigma_+}{2\varepsilon_0} + \frac{\sigma_-}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{Q}{A\epsilon_0}$$

$$V =$$

$$-\frac{dV}{dx} = E$$

$$V_B - V_A = -\int_A^B E\,dx$$



$$\Delta V = V_+ - V_- = -\int_-^+ \underbrace{\vec{E}\cdot d\vec{x}}_{-E\,dx}$$

$$V_{cap} = \Delta V = E\int_0^d dx = Ed$$

units of Electric field $N/C = \frac{Nm}{Cm} = \frac{J}{Cm}$

$$= \frac{J/C}{m} = \frac{V}{m}$$

$$Ed = V$$

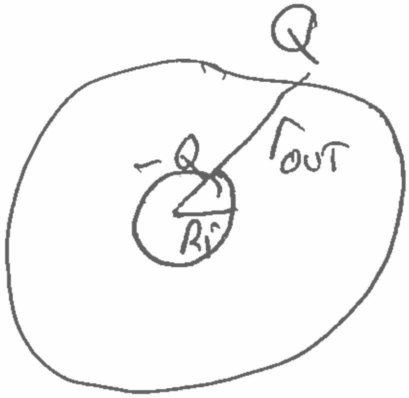
$$\cancel{V = \frac{E}{d}} \qquad E = \frac{V}{d}$$

$$E_{cap} = \frac{Q}{A\,\varepsilon_0} = \frac{V}{d}$$

$$Q = \frac{A\,\varepsilon_0}{d} V \qquad Q = CV$$

$$\Rightarrow C = \frac{A\,\varepsilon_0}{d}$$ Parallel Plate

Cylindrical capacitor

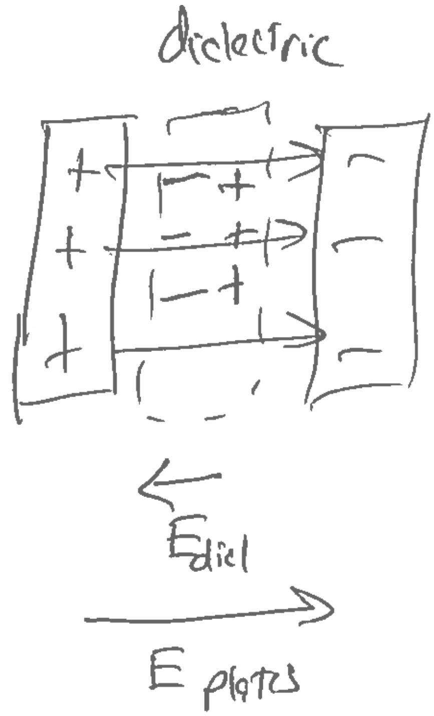


Capacitance depends on Geometry and materials of capacitor

NOT on Voltage or charge.

$$C_{diel} = K C_0$$

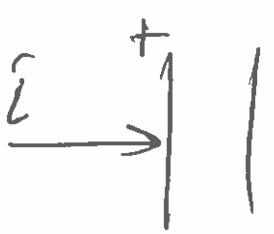
K - dielectric constant.



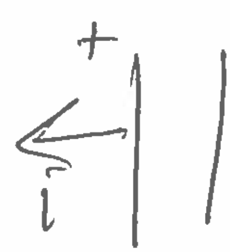
E_{cap} is reduced

$$Q = CV$$

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

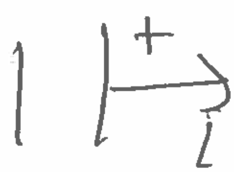


$+i$



$-i$

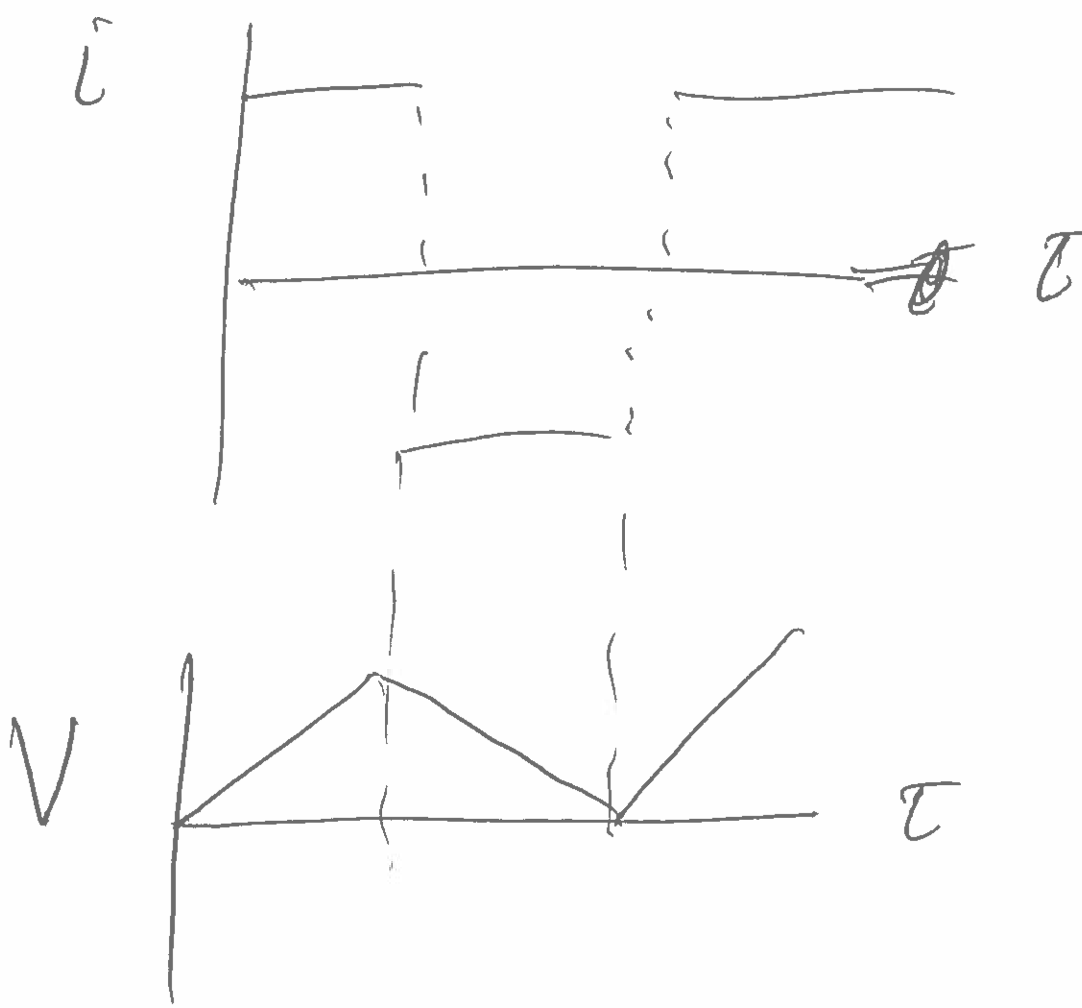
C

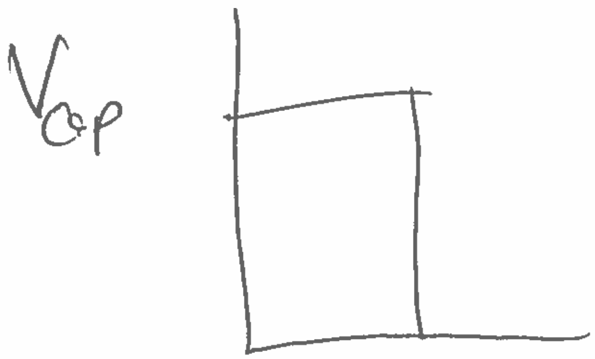


If V is constant

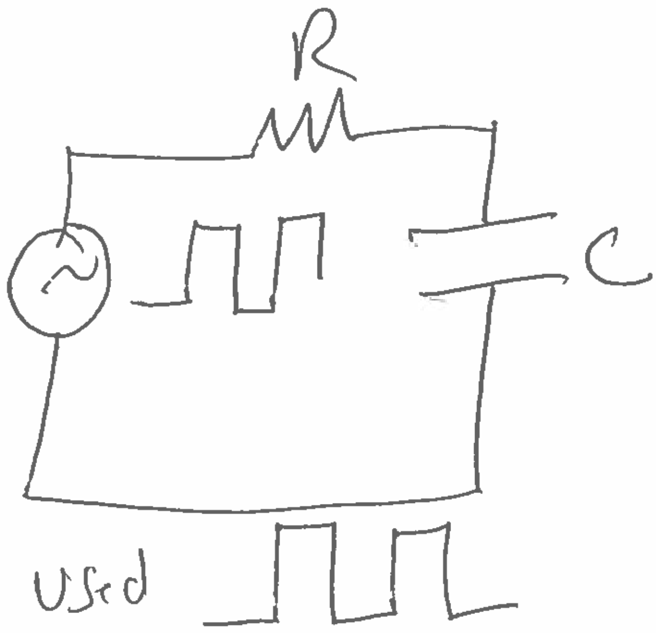
$$i = 0$$

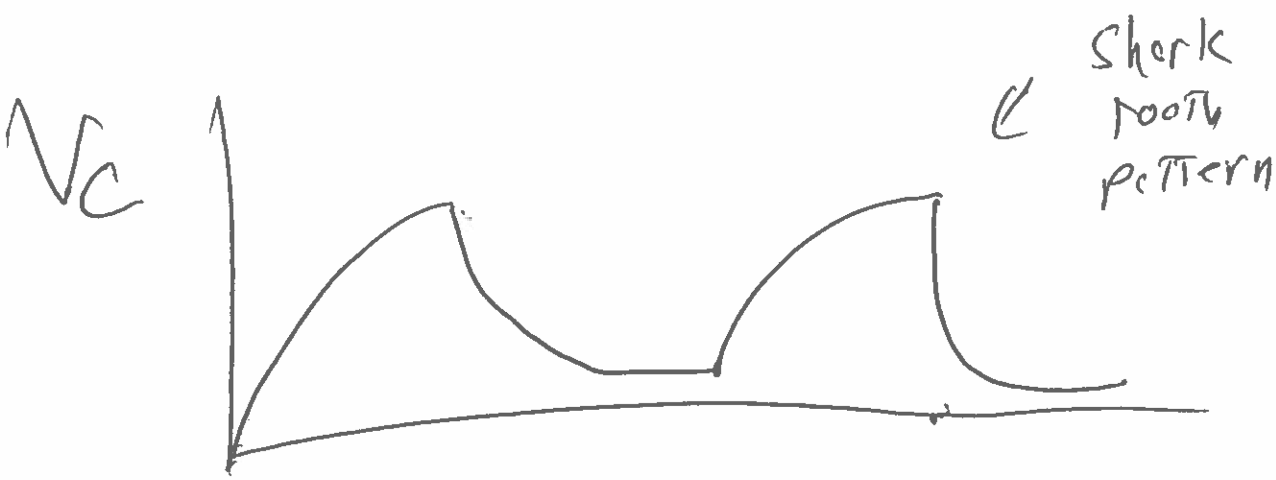
Voltage must be continuous on a capacitor. While current can be discontinuous, voltage cannot be.





can never look so.



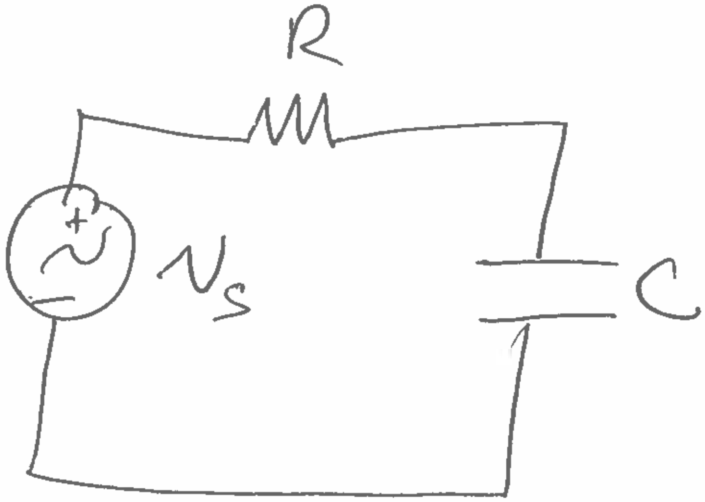


$$v_{cap}(t^-) = v_{cap}(t^+)$$

$t^-, t^+ \quad 0^-, 0^+ \quad 1^-, 1^+$ infinitesimal differences

Voltage across a Capacitor cannot

Change instantaneously



$$V_s = A\sin(\omega t)$$

$$V_c \overset{?}{=} B\sin(\omega t)$$

$$i = C\frac{dV_c}{dt} = C\frac{d}{dt}(B\sin(\omega t))$$

$$i = CB\omega\cos(\omega t)$$

$$\sin(90-\theta) = \cos(\theta)$$

Phase difference between sin and cos

Cos is ahead of sin by $90°$ or $\frac{\pi}{2}$

Current creates voltage on capacitor

So current is ahead in phase of voltage on a capacitor.

Capacitor $i = C\frac{dv}{dt}$ i ahead of V

Resistor $i = \frac{1}{R} V$ i in phase with V

inductor $V_1 = L\frac{di}{dt}$ i behind of V

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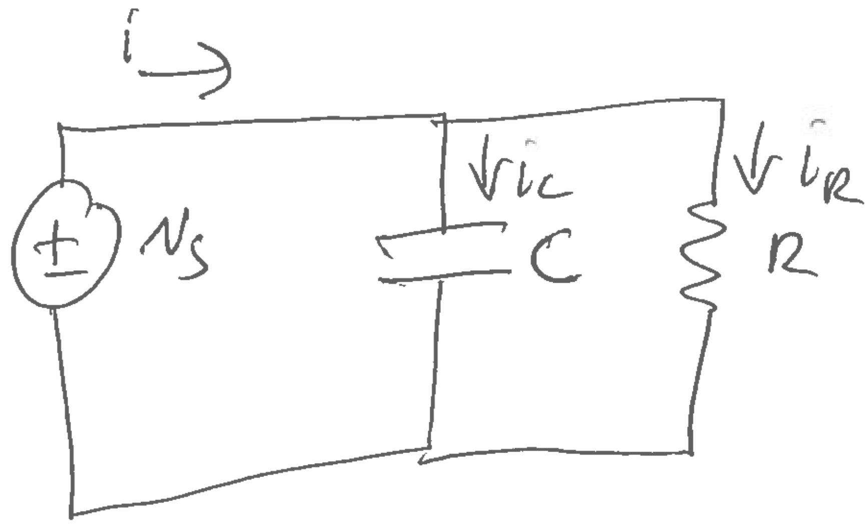
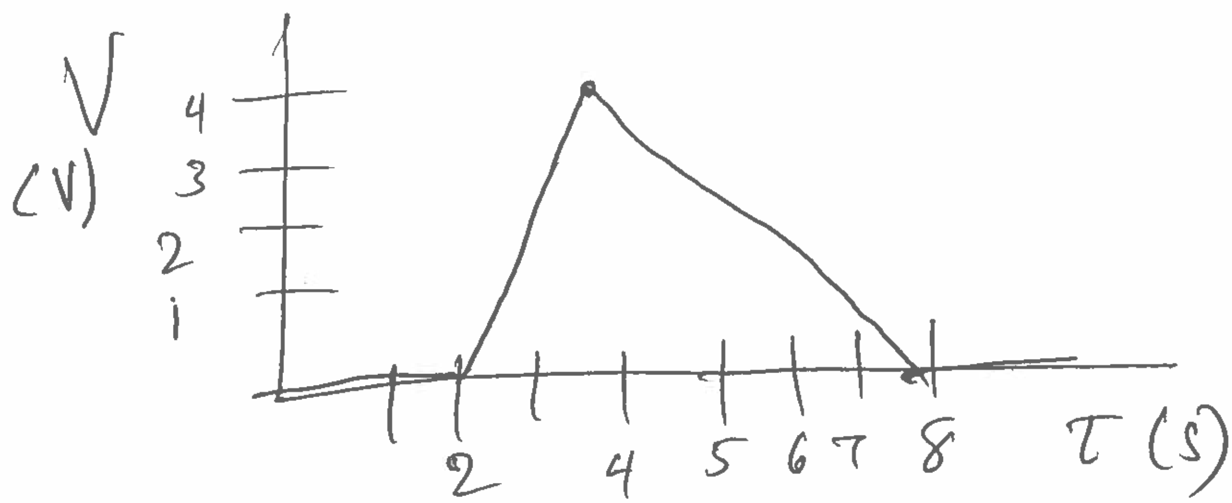
E is emf ⇒ voltage



$C = 1\,F \quad R = 1\,\Omega$

$$i = ? \text{ if } V_s = \begin{cases} 0 & 0 \le t \le 2 \\ 2t-4 & 2 \le t \le 4s \\ 8-t & 4 \le t \le 8s \\ 0 & t \geq 8s \end{cases}$$



$$i = i_c + i_R$$

$$i_R = \frac{V_s}{R} = \begin{cases} 0 & 0 \le t \le 2S \\ 2t-4 & 2 \le t \le 4S \\ 8-t & 4 \le t \le 8S \\ 0 & t \geq 8S \end{cases}$$

$$i_C = C \frac{dV}{dt}$$