Find current, power and energy for a capacitor ($C = 5\,mF$) with a voltage profile shown (on next page) Book ex ~~7-3-2~~ 7-3-2

Use 3 data points

$(0,0)$, $(2,100)$, $(3,50)$

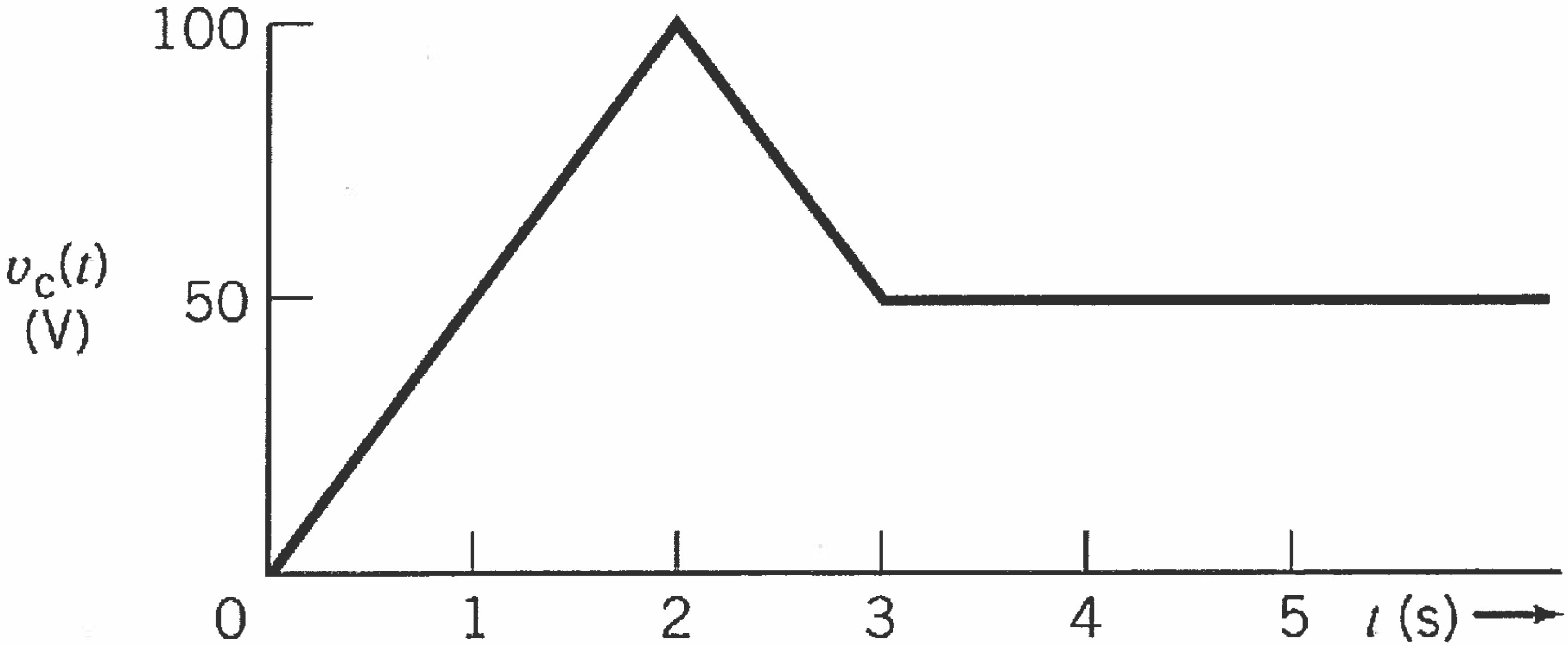
$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$y - 0) = \frac{(100 - 0)}{(2 - 0)}(x - 0)$$

$$\Rightarrow y = 50x$$

$$(y - 100) = \frac{(50 - 100)}{3 - 2}(x - 2)$$

$$y = -50x + 200$$



07-03-03

$$v_c(t) = \begin{cases} 50t \text{ V} & 0 \le t \le 2\text{s} \\ -50t + 200 \text{ V} & 2\text{s} \le t \le 3\text{s} \\ 50 \text{ V} & t \ge 3\text{s} \end{cases}$$

3

$$i_c = C\frac{dv_c}{dt} = 5\text{mF}\begin{cases} \frac{d}{dt}(50t) & 0 \le t \le 2 \\ \frac{d}{dt}(-50t + 200) & 2 \le t \le 3 \\ 0 & t \ge 3 \end{cases}$$

$$i_c = \begin{cases} 5\times10^{-3}(50) \text{ mA} = 0.25\text{A} & 0 \le t \le 2 \\ 5\times10^{-3}(-50) \text{ mA} = -0.25\text{A} & 2\text{s} \le t \le 3\text{s} \\ 5\times10^{-3}(0) = 0\text{A} & t \ge 3\text{s} \end{cases}$$

Power $= i v$

$$P = \begin{cases} 0.25A\,(50t) \\ -0.25A\,(-50t + 200) \\ 0\quad(50) \end{cases} = \begin{cases} 12.5t\ W & 0 \le t \le 2s \\ 12.5t - 50\ W & 2s \le t \le 3s \\ 0\ W & t \ge 3s \end{cases}$$

$$W \Rightarrow \quad W(t) - W(a) = \int_a^t P(t)\,d\tau$$

$0 \le t \le 2s$

$$W(t) - W(0s) = \int_0^t 12.5t\ d\tau = 6.25t^2 - 0$$

$$W(t) = 6.25\,t^2\ (J) \quad 0 \le t \le 2s$$

$W(0) = 0$ ✓

$W(2) = 6.25(2)^2 = 25J$

$2 \le \tau \le 3\,s$

$$w(t) - w(2) = \int_{2}^{\tau} 12.5\tau - 50 \, d\tau$$

$$w(\tau) - 25J = 6.25t^2 \Big|_{2}^{\tau} - 50\tau \Big|_{2}^{\tau}$$

$$w(t) = 25J + 6.25t^2 - \underbrace{25J}_{6.25(2)^2} - 50\tau + \underbrace{50(2)}_{100}$$

$$w(t) = 6.25t^2 - 50\tau + 100J$$

$$w(3) = 6.25(3)^2 - \underbrace{50(3) + 100J}_{-50J}$$

~~43.75~~

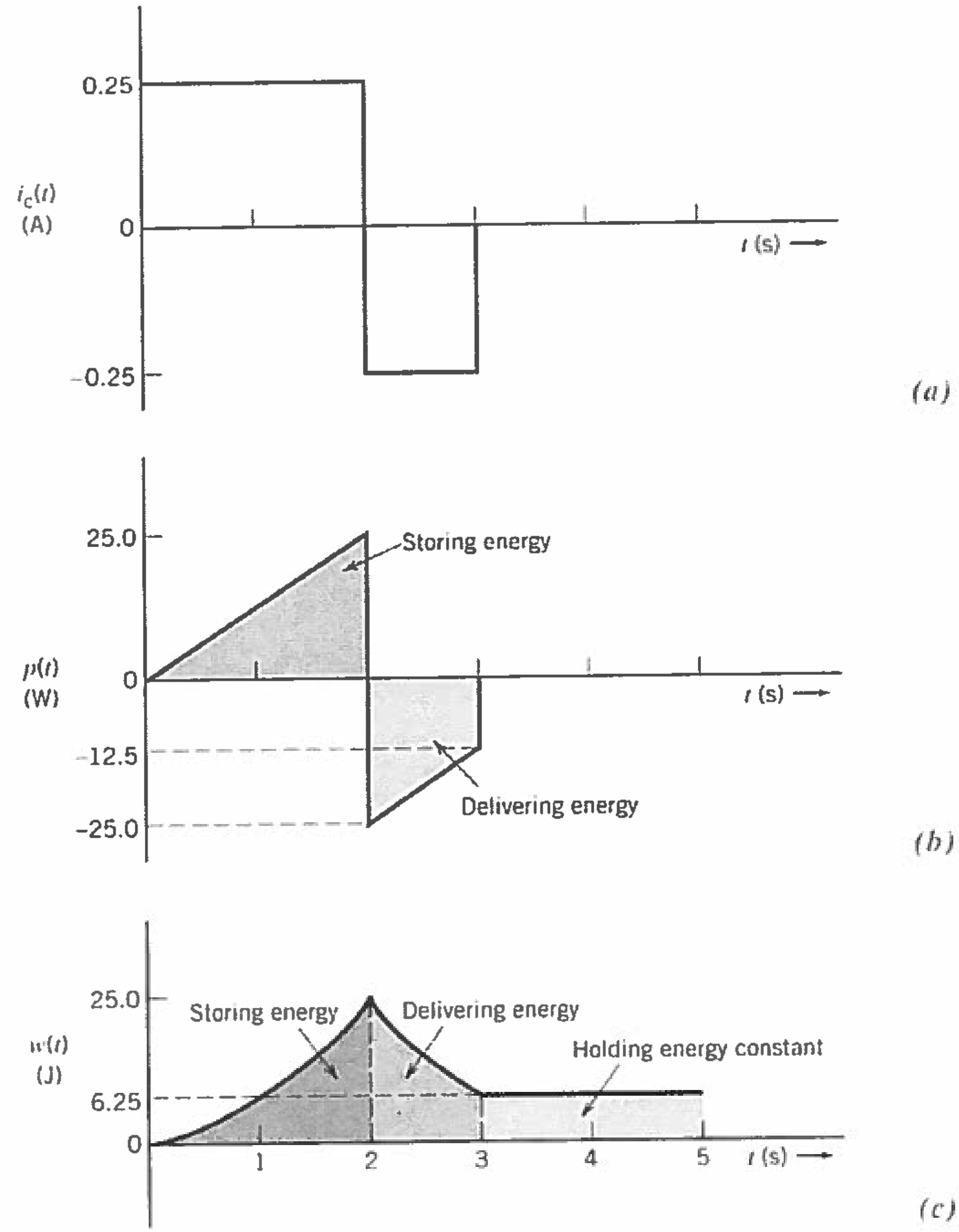
[illegible] 56.25J

$$\underline{w(3) = 6.25J}$$

$$w(t) - w(3) = \int_3^{\tau} 0 \, d\tau = 0$$

$$w(t) = w(3) = 6.25\ \text{J} \quad \text{for } t \geq 3\text{s}$$

$$w(t) = \begin{cases} 6.25t^2\ \text{J} & 0 \leq t \leq 2\text{s} \\ 6.25t^2 - 50t + 100\ \text{J} & 2\text{s} \leq t \leq 3\text{s}, \\ 6.25\ \text{J} & t \geq 3\text{s} \end{cases}$$

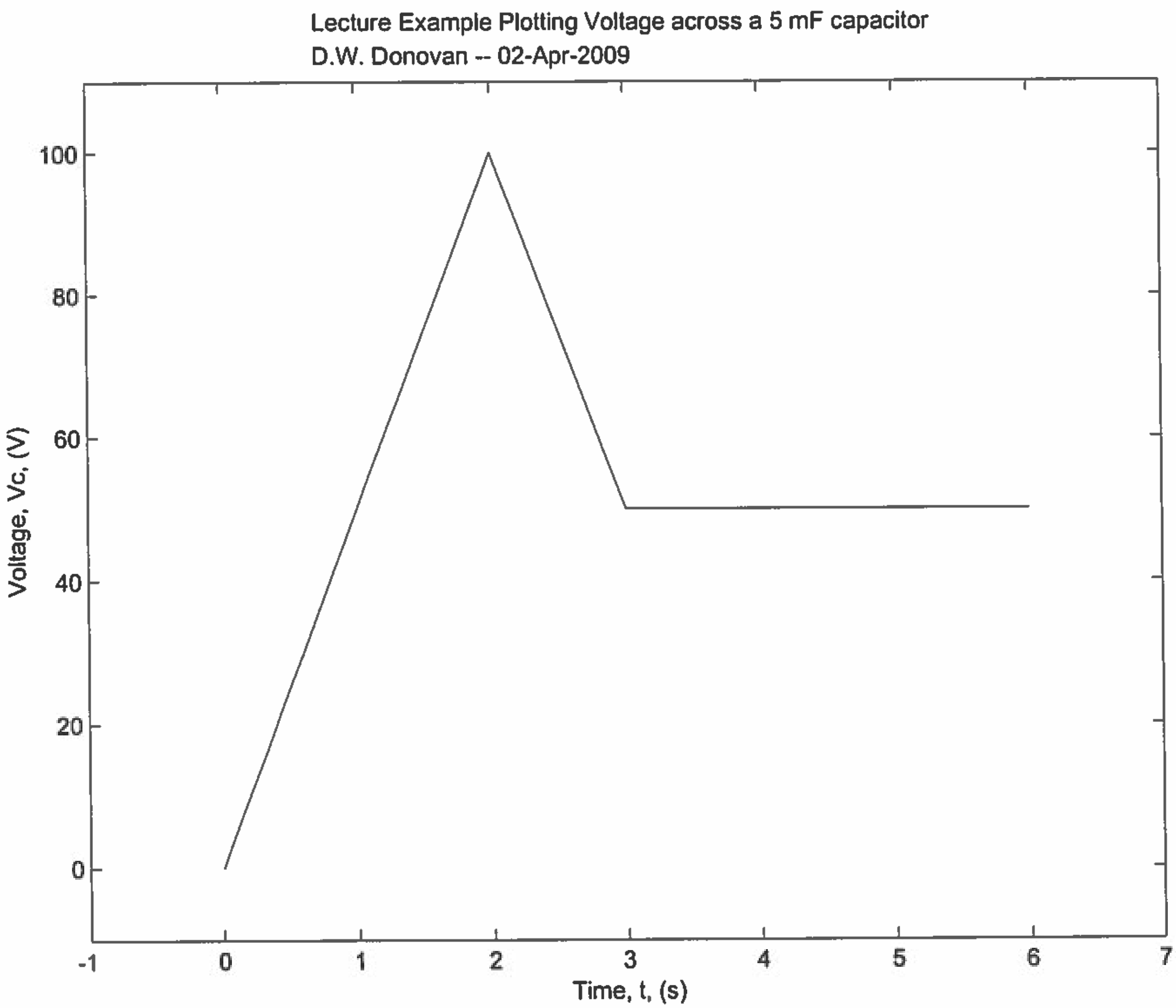


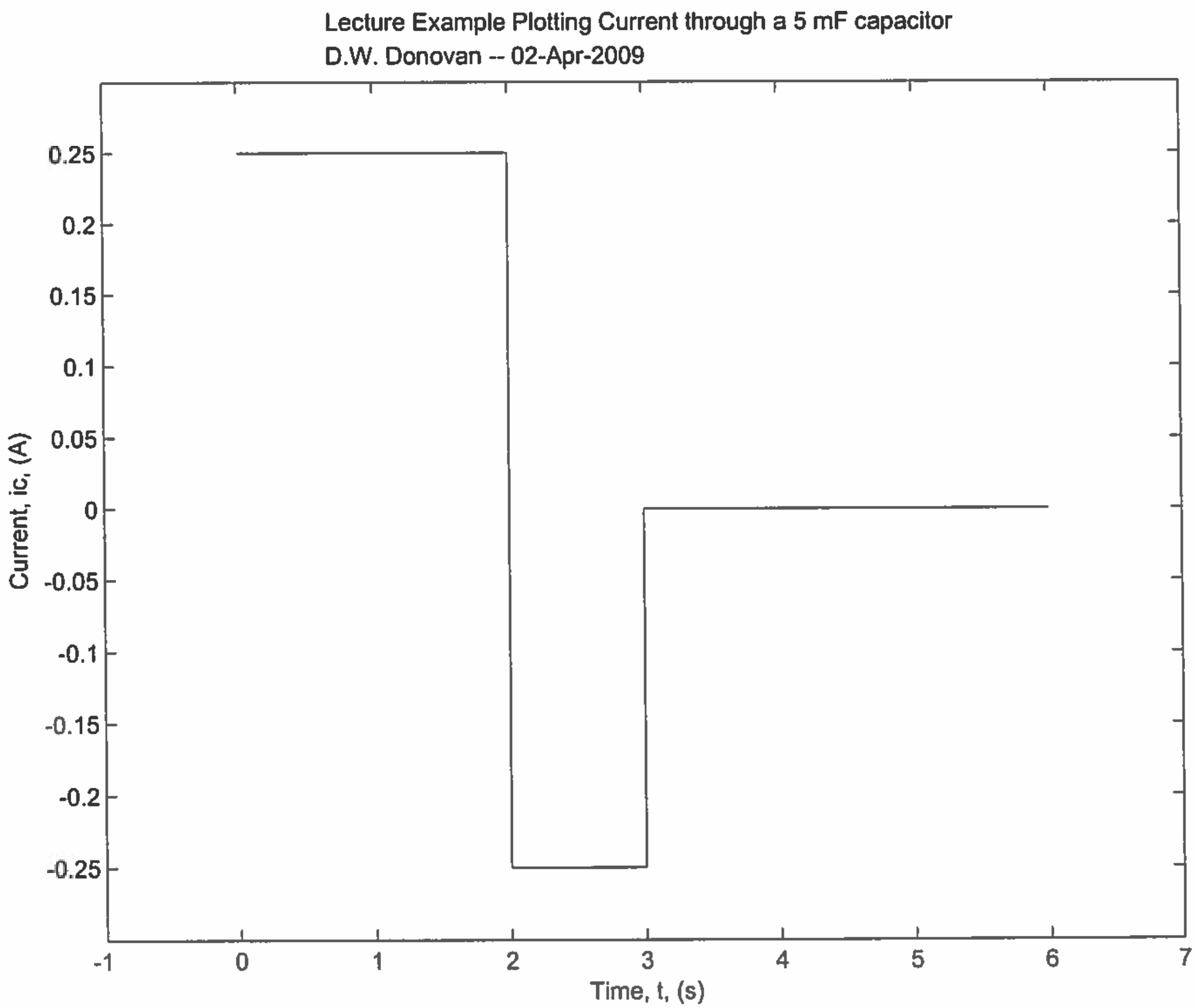
0.25
$i_C(t)$
(A)
0
−0.25
t (s)
(a)
25.0
Storing energy
$p(t)$
(W)
0
−12.5
−25.0
Delivering energy
t (s)
(b)
25.0
Storing energy
Delivering energy
Holding energy constant
$w(t)$
(J)
6.25
0
1
2
3
4
5
t (s)
(c)

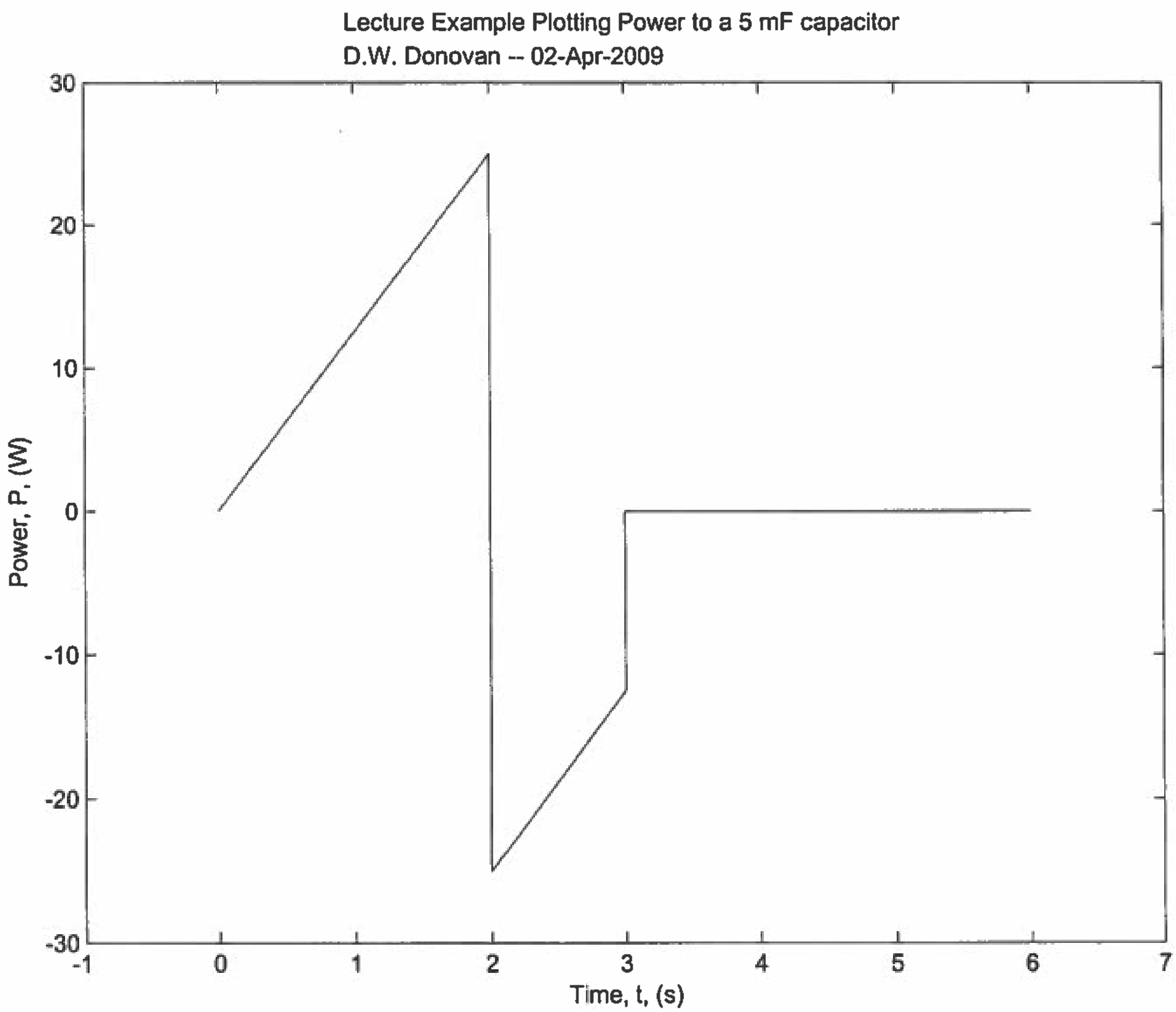
07-03-04

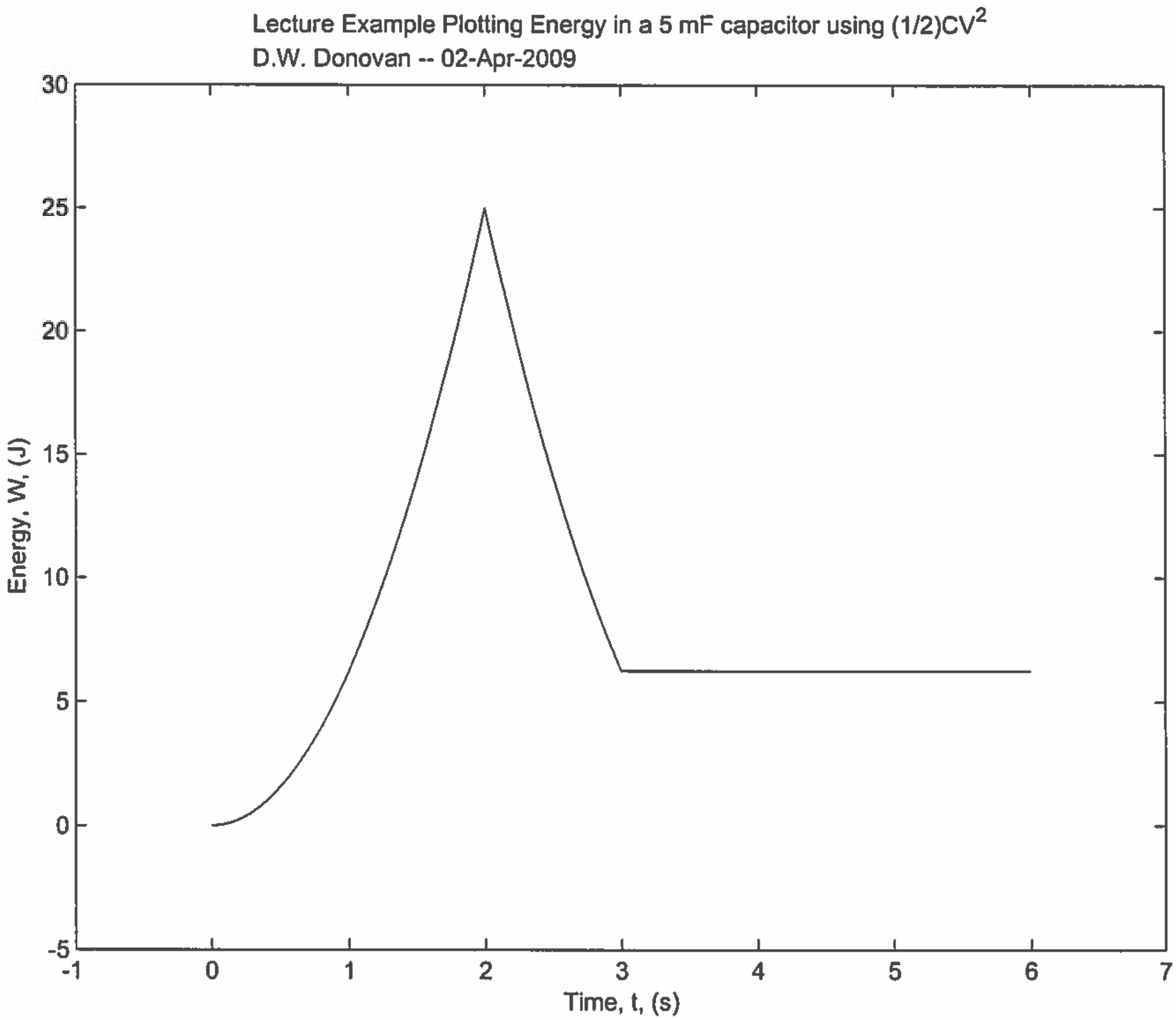
```
%Program to solve a lecture example
%version 2009-04-02 DW Donovan
clear all;

C=0.005;

t1=[0:0.01:2]';
t2=[2:0.01:3]';
t3=[3:0.01:6]';
t=[t1;t2;t3];

v1=50*t1;
v2=-50*t2+200;
v3=50*t3./t3;
v=[v1;v2;v3];

i1=C*(50)*t1./t1;
i2=C*(-50)*t2./t2;
i3=C*(0)*t3./t3;
ii=[i1;i2;i3];

p1=12.5*t1;
p2=12.5*t2-50;
p3=0*t3./t3;
p=[p1;p2;p3];

wc1=0.5*C*v1.^2;
wc2=0.5*C*v2.^2;
wc3=0.5*C*v3.^2;
wc=[wc1;wc2;wc3];

wi1=6.25*t1.^2;
wi2=6.25*t2.^2-50*t2+100;
wi3=6.25*t3./t3;
wi=[wi1;wi2;wi3];

figure
plot (t,v,'k')
ttv='Lecture Example Plotting Voltage across a 5 mF capacitor';
tn='D.W. Donovan -- ';
tttv=[ttv '\newline' tn date];
title(tttv)
xlabel('Time, t, (s)')
ylabel('Voltage, Vc, (V)')
axis([-1 7 -10 110])

figure
plot(t,ii,'k')
tti='Lecture Example Plotting Current through a 5 mF capacitor';
tn='D.W. Donovan -- ';
ttti=[tti '\newline' tn date];
title(ttti)
```

```
xlabel('Time, t, (s)')
ylabel('Current, ic, (A)')
axis([-1 7 -0.3 0.3])

figure
plot(t,p,'k')
ttp='Lecture Example Plotting Power to a 5 mF capacitor';
tn='D.W. Donovan -- ';
tttp=[ttp '\newline' tn date];
title(tttp)
xlabel('Time, t, (s)')
ylabel('Power, P, (W)')
axis([-1 7 -30 30])

figure
plot(t,wc,'k')
ttwc='Lecture Example Plotting Energy in a 5 mF capacitor using (1/2)CV^{2}';
tn='D.W. Donovan -- ';
tttwc=[ttwc '\newline' tn date];
title(tttwc)
xlabel('Time, t, (s)')
ylabel('Energy, W, (J)')
axis([-1 7 -5 30])

figure
plot(t,wi,'k')
ttwi='Lecture Example Plotting Energy in a 5 mF capacitor using integral of Power';
tn='D.W. Donovan -- ';
tttwi=[ttwi '\newline' tn date];
title(tttwi)
xlabel('Time, t, (s)')
ylabel('Energy, W, (J)')
axis([-1 7 -5 30])
```

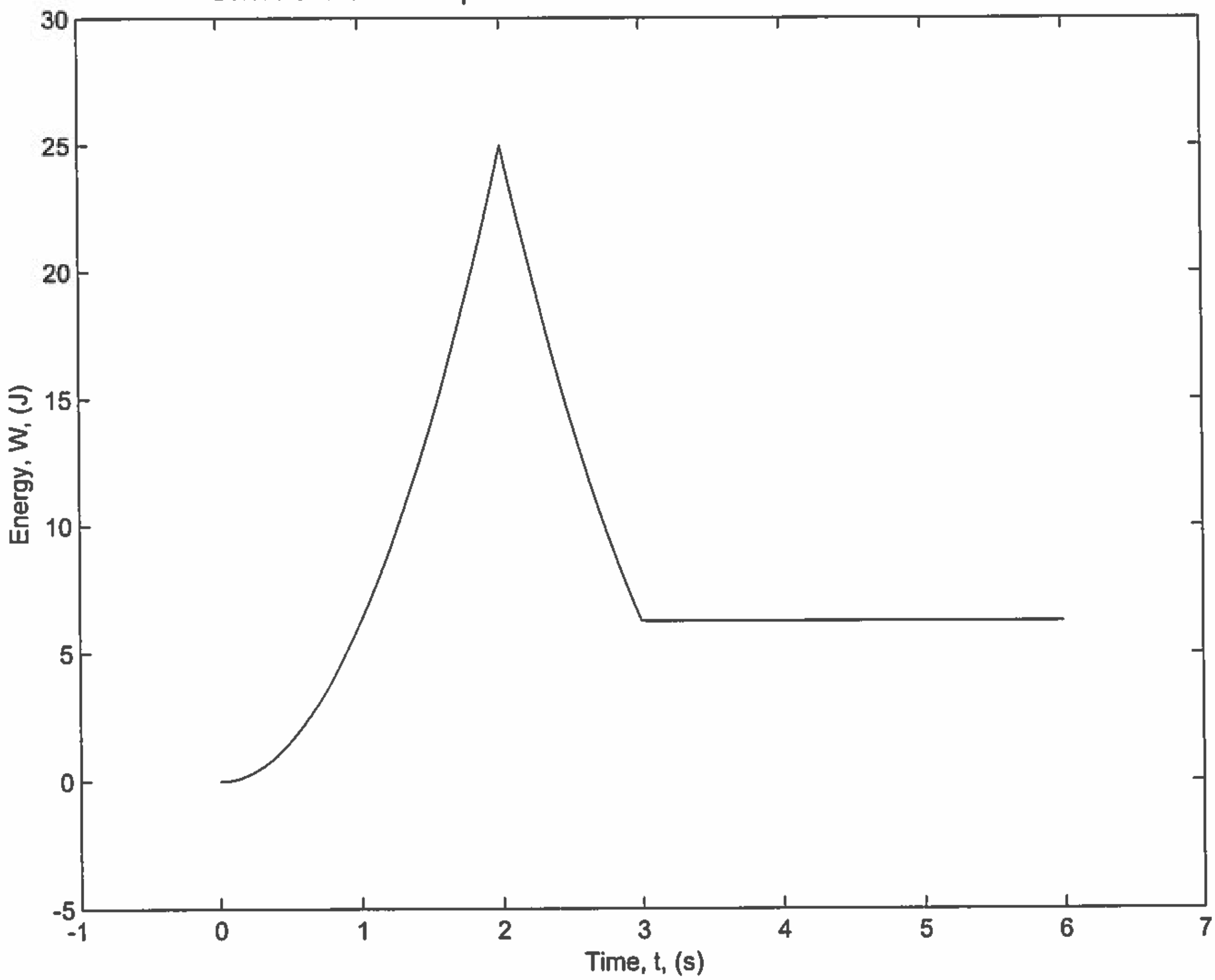

Lecture Example Plotting Voltage across a 5 mF capacitor
D.W. Donovan -- 02-Apr-2009
Voltage, Vc, (V)
Time, t, (s)
100
80
60
40
20
0
-1
0
1
2
3
4
5
6
7


Lecture Example Plotting Current through a 5 mF capacitor
D.W. Donovan -- 02-Apr-2009
Current, ic, (A)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-1
0
1
2
3
4
5
6
7
Time, t, (s)


Lecture Example Plotting Power to a 5 mF capacitor
D.W. Donovan -- 02-Apr-2009
30
20
10
0
-10
-20
-30
-1
0
1
2
3
4
5
6
7
Power, P, (W)
Time, t, (s)


Lecture Example Plotting Energy in a 5 mF capacitor using (1/2)CV^2
D.W. Donovan -- 02-Apr-2009
Energy, W, (J)
Time, t, (s)
30
25
20
15
10
5
0
-5
-1
0
1
2
3
4
5
6
7

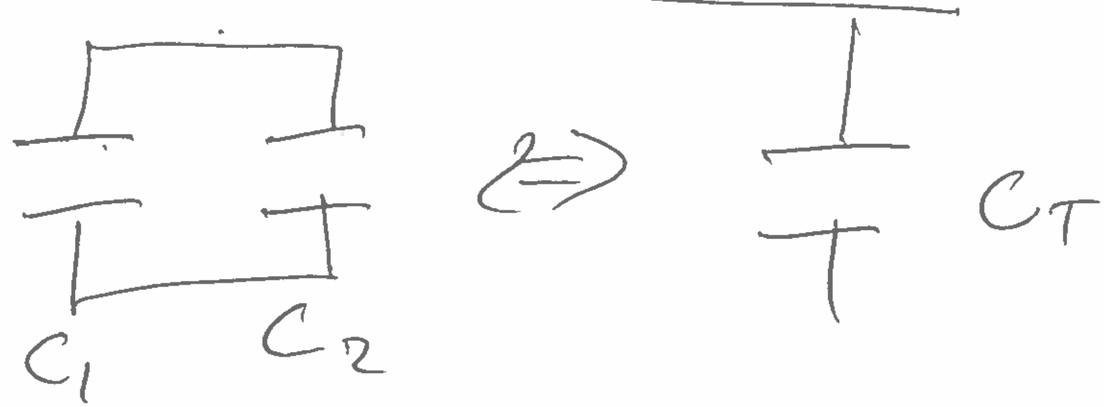
Lecture Example Plotting Energy in a 5 mF capacitor using integral of Power
D.W. Donovan -- 02-Apr-2009



$i \Rightarrow Q = \int i t$

$$V = \frac{Q}{C} = \frac{1}{C} \int i \, d\tau$$

capacitors in series / parallel

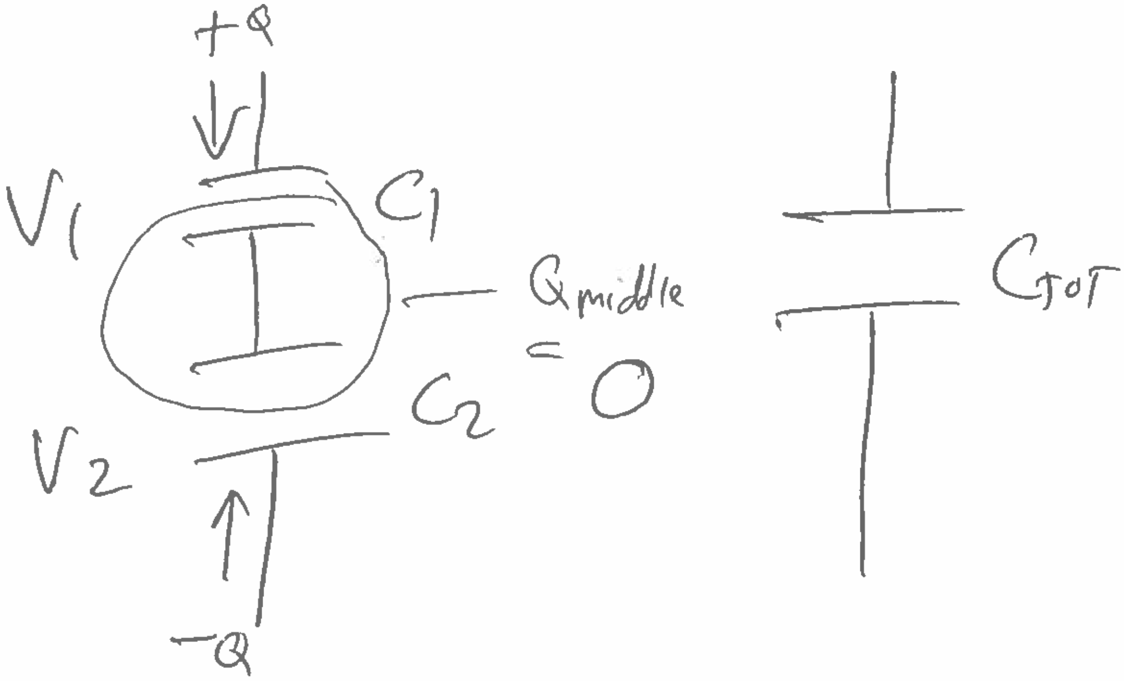


$V_1 = V_2 = V_{eff}$ Voltages must equilibrate

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{eff}}{C_{eff}}$$

$$Q_1 + Q_2 = C_1 V + C_2 V \Rightarrow (C_1 + C_2) V$$

$C_{TOT} = C_1 + C_2 + \cdots + C_n$ Parallel



$$V_{TOT} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{TOT}}$$

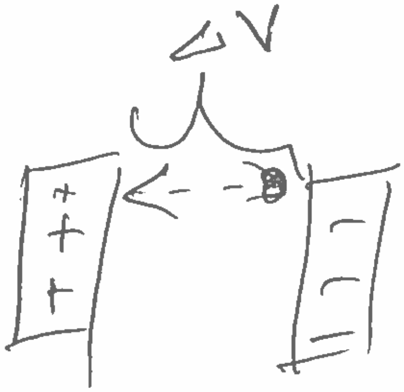
$$\frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

~~Par~~ Series

Inductors are typically coils
That induce currents due to Faraday's Law

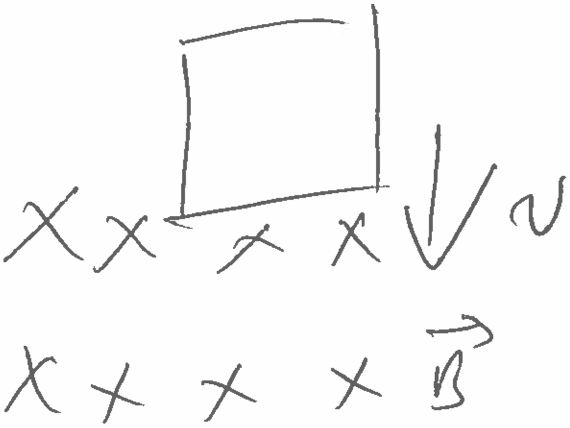
Faraday's Law $\quad \mathcal{E} = -N \frac{d\Phi_B}{dt}$

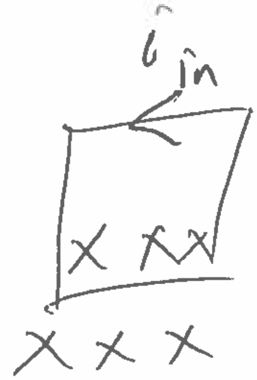
Emf $\mathcal{E}$ is similar but different
from voltage



$\Delta V > 0$ implies going from − plate
to positive plate

$\vec{E}$ goes from + → −





which way is i_{ind}.

magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$

is increasing

So induced field wants to

oppose increase in Φ_B so

$B_{induced}$ points out of paper ⊙

now i_{ind} to create B_{ind}

must go counterclockwise