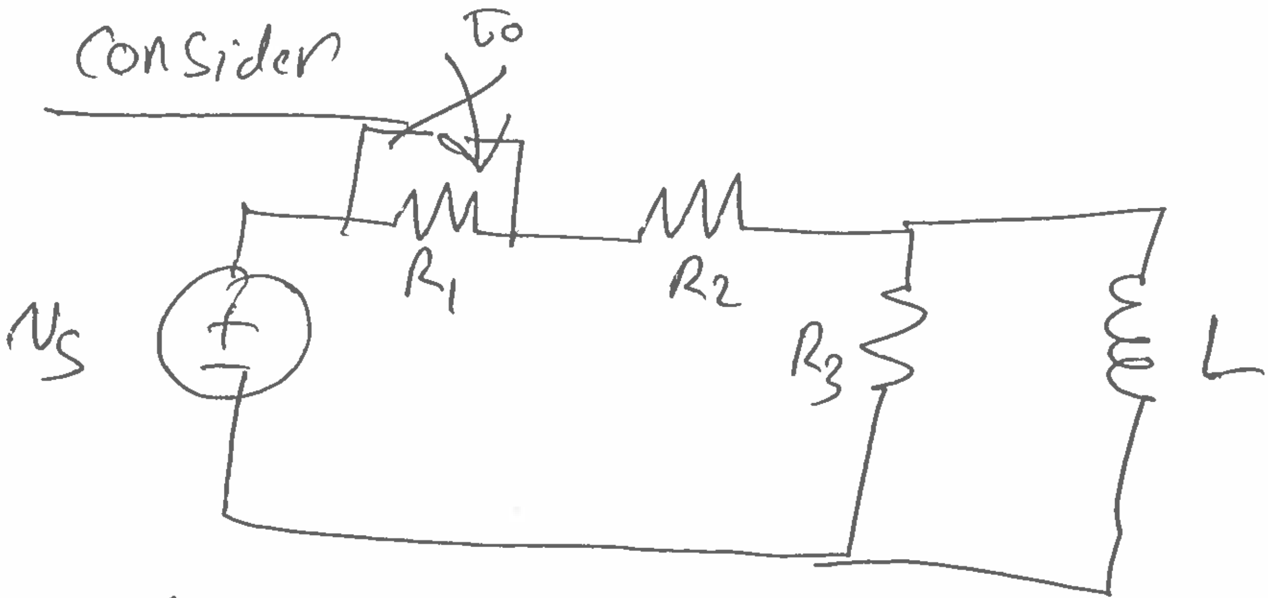
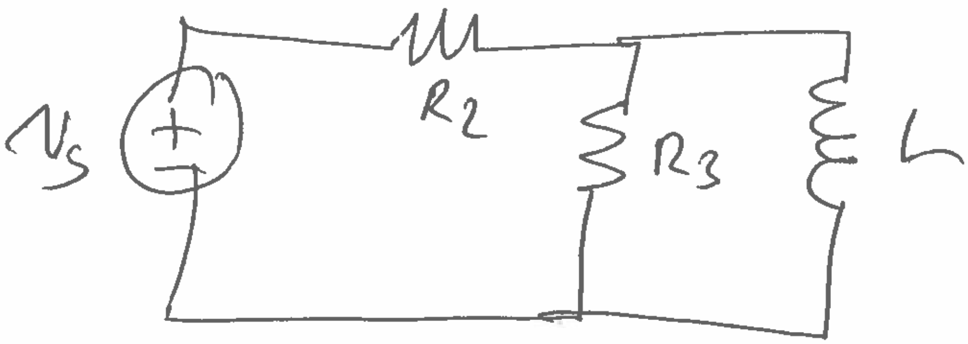
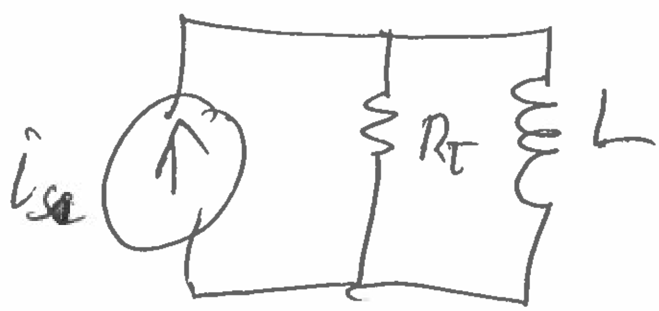
consider



after switch thrown





$$i_s = \frac{v_s}{R_2} \qquad R_T = \frac{R_2 R_3}{R_2 + R_3}$$

$$i_s = i_R + i_L$$

$$i_R = \frac{V_L}{R_T} \qquad V_L = L\frac{di}{dt}$$

$$i_R = \frac{L}{R_T}\frac{di_L}{dt}$$

$$i_S = \frac{L}{R_t}\frac{di_L}{dt} + i_L$$

$$\cancel{\frac{di_L}{dt} = \frac{R_t}{L} i_S -}$$

$$\frac{di_L}{dt} + \frac{R_t}{L} i_L = \frac{R_t}{L} i_S$$

$$\tau_{L} = \frac{L}{R}$$

$$\frac{di_L}{dt} + \frac{1}{\tau} i_L = \frac{i_s}{\tau} = K$$

$$\boxed{\frac{dx}{dt} + \frac{x(t)}{\tau} = K}$$

– same form as capacitor on Monday

$$\frac{dx(t)}{dt} = K - \frac{x(t)}{\tau}$$

$$\frac{dx(t)}{dt} = \frac{K\tau - x(t)}{\tau}$$

$$\frac{dx(t)}{(K\tau - x(t))} = \frac{dt}{\tau}$$

$$\int \frac{dx(t)}{x(t) - k\tau} = \int -\frac{dt}{\tau}$$

$$\ln(x - k\tau) = -\frac{t}{\tau} + D$$

$\uparrow$ integration constant

$$e^{\ln(x - k\tau)} = e^{\left(-t/\tau + D\right)}$$

$$x - k\tau = e^{D} e^{-t/\tau}$$

$$\boxed{x(t) = k\tau + A e^{-t/\tau}}$$

initial conditions $\Rightarrow x(0)$ at $t = 0$

$$x(0) = k\tau + A e^{0} = k\tau + A$$

$$x(0) = k\tau + A$$

at $t = \infty$

$$X(\infty) = K\tau + A e^{-\infty/\tau} = K\tau$$

$$X(t) = K\tau + A e^{-t/\tau}$$

$$K\tau = X(\infty)$$

$$X(0) = K\tau + A \Rightarrow A = X(0) - \overset{K\tau}{X(\infty)}$$

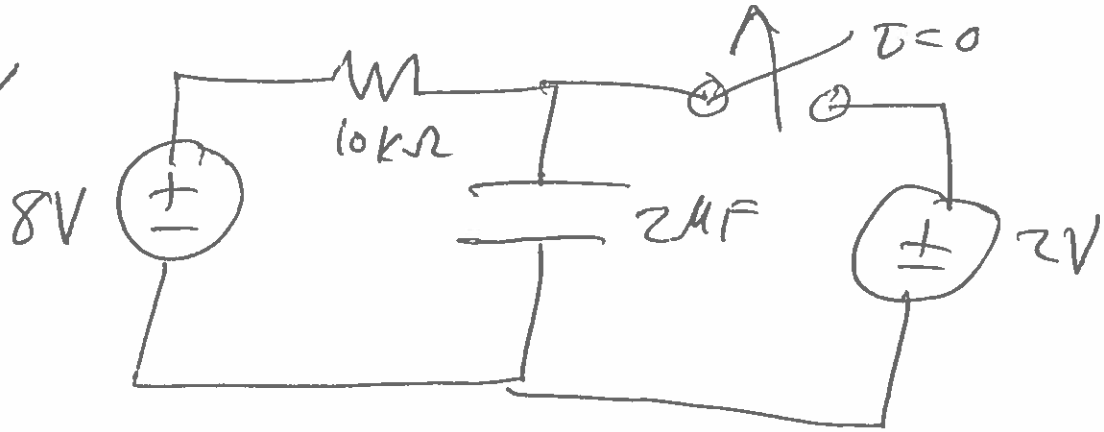
$$A = X(0) - X(\infty) \qquad K\tau = X(\infty)$$

$$\boxed{X(t) = X(\infty) + \left(X(0) - X(\infty)\right) e^{-t/\tau}}$$

long term steady state solution (Forced)

Transient solution (natural)

eg/



$v_c(t) = ?$

at $t = 0$ $\quad v_c(0) = 2V$

at $t = \infty$ $\quad V_c(\infty) = 8V$

$R_t = 10k\Omega$

$$v(t) = 8V + (2V - 8V)e^{-t/(10k\Omega)(2\mu F)}$$

$$v(t) = 8V - 6V e^{-t/0.20s}$$

$t = 0 \Rightarrow v(t) = 8 - 6V e^{0} = 2V$ ✓

$t = \infty \Rightarrow v(t) = 8V - 6V e^{-\infty} = 8V$ ✓

If we have non constant power sources

$$\frac{dx(t)}{dt} + a\,x(t) = y(t)$$

$a = \frac{1}{\tau}$ Previously $y(t) = K$

Integrating factor $e^{\int a\,d\tau} = e^{a\tau}$

$$\frac{dx(t)}{dt} e^{a\tau} + a x e^{a\tau} = y e^{a\tau}$$

$$\frac{d}{dt}(x e^{a\tau}) = y e^{a\tau}$$

$$\int d(x e^{a\tau}) = \int y e^{a\tau} d\tau$$

$$x e^{at} = \int y(t) e^{at} \, dt + K$$

$$x(t) = e^{-at} \int y(t) e^{at} \, dt + K e^{-at}$$

Check if $y(t) = M$ = constant

$$x(t) = \underbrace{e^{-at} \int M e^{at} \, dt}_{e^{-at} \frac{M}{a} e^{at}} + K e^{-at}$$

$$x(t) = \frac{M}{a} + K e^{-at} \qquad a = \frac{1}{\tau}$$

$$x(t) = M\tau + K e^{-t/\tau}$$

$$x(t) = x(\infty) + (x(0) - x(\infty)) e^{-t/\tau}$$

$y(t) \neq$ constant

$$X(t) = X_F + X_n$$

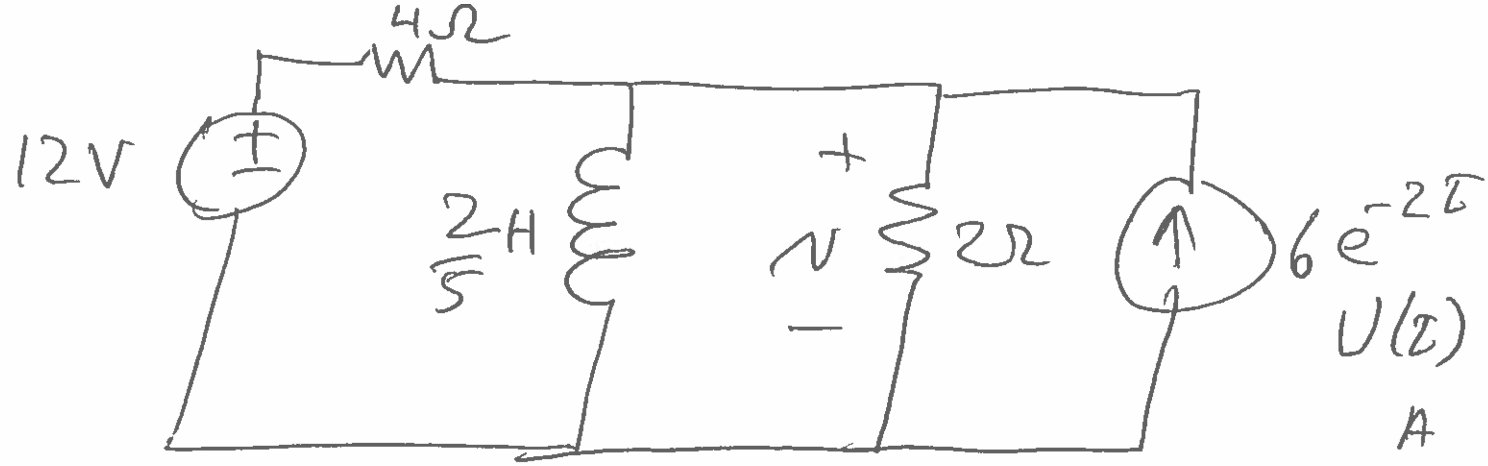
$$X(t) = \underbrace{e^{-at} \int y(t)\, e^{at}\, dt}_{X_F} + \underbrace{K e^{-t/\tau}}_{X_n}$$

typical non constant Forcing functions

are sin, cos, e

Use $X_F = A\cos(wt) + B\sin(wt)$

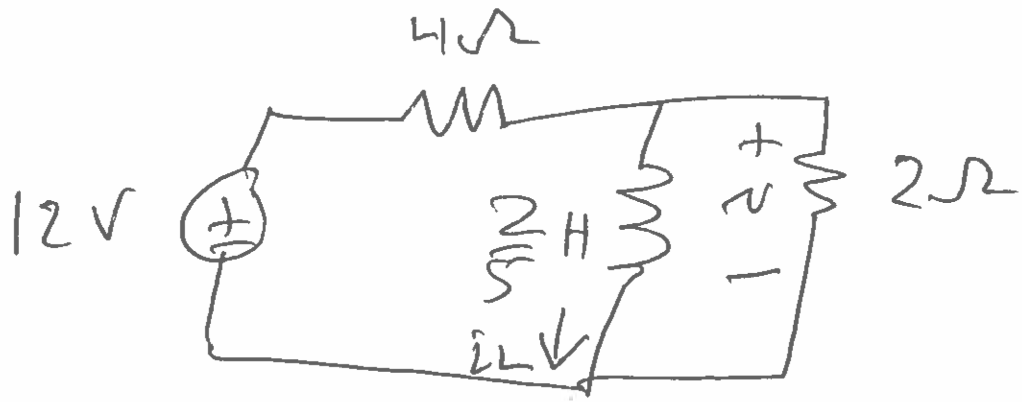
solve diff-eq and Apply Boundery or time conditions.



$U(t-t_0) = 0 \quad t < t_0 \quad = 1 \quad t \geq t_0$

↑
step function

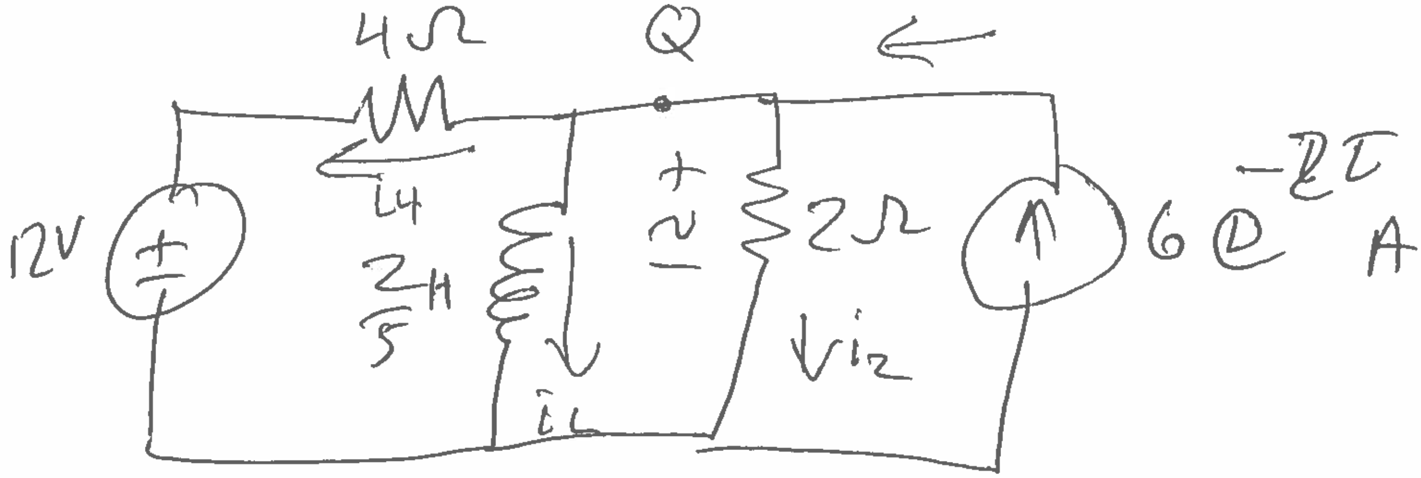
$t < 0$ Circuit looks like



$$i_L(0^-) = \frac{12V}{4\Omega} = 3A$$

after $t = 0$

Circuit looks like



$$i_4 = \frac{V - 12V}{4\Omega} \qquad i_2 = \frac{V}{2\Omega}$$

current at Q

$$6e^{-2t} - i_2 - i_4 - i_L = 0$$

$$6e^{-2t} - \frac{V}{2\Omega} - \frac{V - 12V}{4\Omega} - i_L = 0$$

$$V = L\frac{di_L}{dt}$$

$$6e^{-2t} - \frac{L}{2\Omega}\frac{di_L}{dt} - \frac{L}{4\Omega}\frac{di_L}{dt} + 3 - i_L = 0$$

$$6e^{-2t} - \frac{2/5}{2\Omega}\frac{di_L}{dt} - \frac{2/5}{4\Omega}\frac{di_L}{dt} + 3 - i_L = 0$$

$$\frac{2}{10}\frac{di_L}{dt} + \frac{1}{10}\frac{di_L}{dt} + i_L = 6e^{-2t} + 3$$

$$\frac{di_L}{dt} + \frac{10}{3} i_L = 20e^{-2t} + 10$$

$$i_N = A e^{-(10/3)t}$$

$$i_F = B + C e^{-2t}$$

$$i_L = A e^{-(10/3)t} + B + C e^{-2t}$$

$$\frac{di_L}{dt} = -\frac{10}{3} A e^{-10/3 t} - 2C e^{-2t}$$

$$\underline{i(t) = -15 e^{-(10/3)t} + 3 + 15 e^{-2t}}$$