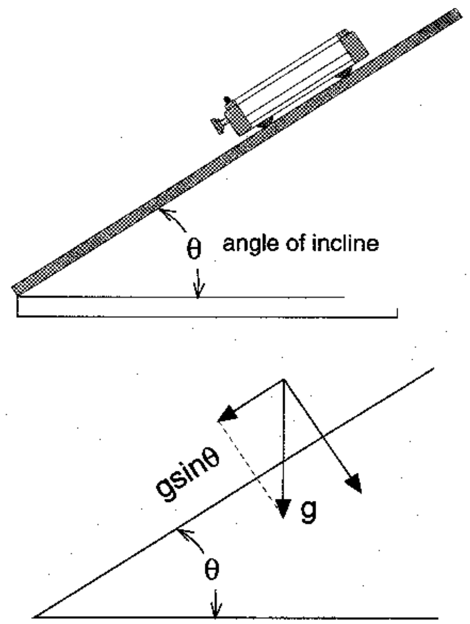
Constant Acceleration

In this lab we examine the motion of an object moving with constant acceleration in one dimension. We use a cart with low-friction wheels running down an inclined plane. We will find the acceleration of the cart using two different graphing techniques. The two values found for the acceleration should be similar, but we don't expect them to be *exactly* the same.



If the acceleration is constant, the following relationship holds

$$\Delta x = v_0 t + \frac{1}{2} a t^2 ,$$

where Δx is the displacement down the incline, v_0 is the initial velocity, *a* is the acceleration, and *t* is the elapsed time.

If the cart on the incline is released from rest, then $v_0 = 0$ and we can write this equation as:

$$\Delta x = \frac{1}{2} a t^2$$

This is the relationship we are going to test in this lab.

Experimental Procedure

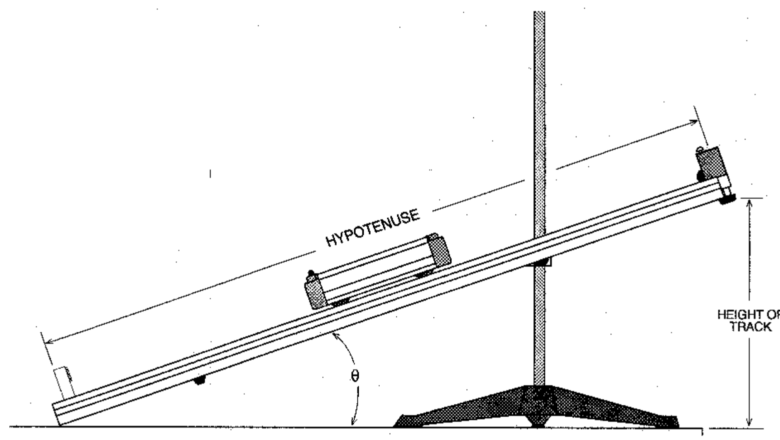
The experimental apparatus is shown below. The angle of inclination is greatly exaggerated. The dynamics cart will be placed onto its track at various marks and released from rest. A stopwatch will be used to measure the time it takes for the cart to travel down the track until the front of it reaches the 110 cm mark.

To increase the accuracy of your measurements, consider the following as you perform this experiment:

1. Practice. It may look easy, but especially at shorter distances, coordinating the cart release and starting the timer takes a few tries. Whenever, you are performing a new experiment, it is always better to do a few practice runs that you do not count in the experiment. These practice runs will also help point out some difficulties that may not be obvious at first.

2. The cart is not a point. We shall use the front of the cart as our reference point. At the bottom of the track is a magnetic bumper which will keep the cart from going off the track. This bumper will stop the cart and push it back up the track. Our end reference point will therefore be the 110 cm mark. Time until the cart's front edge passes this point.

3. Even after a few practice runs, measuring one time for each distance is not the most accurate way to conduct this experiment. For each distance, make four runs and record each time. We shall use the average time for each distance in our analysis.



4. The data will be more accurate if the same person who releases the cart also does the timing. Having a second person time while the other releases the cart introduces an additional reaction time to the measurement. However, one person should not take all the data. Have each partner take two of the four data runs for each distance. This will minimize reaction time differences. Having one person do all four for one distance, and the other do all four for the next, could introduce more error in the data.

5. Start with the 90 cm distance and work down to 10 cm at the end. Since as you work towards 10 cm, your times will get shorter and therefore harder to measure, you will be gaining experience before you get to these times. If there is an obvious problem with a data run, such as friction on a wheel, or an error starting the timer, you may throw the run out and do another one. Do not discard a run because you don't think the time is right.

Experimental Analysis

We will create three graphs to analyze the motion of the cart.

Graph #1 - Δx vs t

Since greater distances x take a longer time t, lets plot x versus t (x on the vertical axis and t on the horizontal axis) and see what the relationship looks like. If the data agrees with the theoretical relationship, the plot should be a parabola. You should be able to verify that the plot is not straight.

Create the graph, ensuring that it has the required elements: a title, your name, the date, axes labeled with the name, symbol, and unit of the variable, etc. Since the data does not form a straight line, do not include a best-fit line through the data.

Here are suggested headings for your data table:

distance Δx (cm)	time t_1 (sec)	time t_2 (sec)	time t_3 (sec)	time t_4 (sec)	average time t (sec)
…	…	…	…	…	…

Graph #2 - Linear Analysis

To measure physical quantities from experimental data, it is common to find a way to plot it in the form of a straight line. This can be done if one correctly guesses the form of the equation relating the two variables. Our guess for the correct equation is

$$\Delta x = \frac{1}{2} a t^2 \,.$$

Consider plotting Δx versus t^2 instead of Δx versus t. It should produce a straight line. To see this, give t^2 a new name, u. Then we would plot Δx versus u, and the equation would be:

$$\Delta x = \left(\frac{1}{2} a\right) u$$

Convince yourself that a plot of Δx versus u will be a straight line with slope $a/2$ (and zero intercept on the vertical axis). So the plot of Δx versus t^2 will produce a straight line with slope a/2, and we can find the acceleration of the cart from

$$a = 2(\text{slope}) \,.$$

To find the value of acceleration by this method, we need a data table. Make a new one with the following suggested columns. Paste the first two columns from the previous table.

distance Δx (cm)	average time t (sec)	t^2 (s^2)
…	…	…

Create the graph of Δx versus t^2 and include the required headings, labeling, and so on. It should be a straight line, so insert the best line on the plot and display the equation for this line. An equation produced by software like Excel will not include the required units, so you need to find the units yourself, by looking at the units on the axes of the graph:

unit of slope = (unit on vertical axis)/(unit on horizontal axis)
unit of intercept = unit on vertical axis

Show how you got the acceleration on the hand-in sheet.

Graph #3 Logarithmic Analysis

Using a logarithmic analysis, we can obtain a straight line graph in another way. We will then be able to compare the acceleration obtained by our two methods. The instructor will review how the logarithmic analysis works. Use the space below to write down the steps. The starting point is the guess for the equation relating the two variables:

$$\Delta x = \frac{1}{2} a t^2 .$$

The first step of the logarithmic analysis is always to take the natural logarithm of both sides:

You should get to the form:

$$[\ln(\Delta x)] = 2[\ln t] + \ln(a/2)$$

Compare it with the equation of a straight line:

$$y = mx + b$$

Convince yourself that a plot of ln(Δx) versus ln t is a straight line with slope 2 and intercept ln(a/2). This means that the acceleration can be found from the intercept b.

To create a graph of ln(Δx) versus ln t, you will need a new data table, with columns for the values of ln(Δx) and ln t. Note that for the logarithmic function, the answer has no unit. So for example, if you want ln of .33 meters, you have ln(length /meter) = ln(.33) = -1.12, with no unit. Here are suggested columns for the table. Paste the first two columns from your previous table.

distance Δx (cm)	average time t (sec)	ln(Δx/cm)	ln(t/sec)
…	…	…	…

Create the graph and include the usual headings, labeling, and so on. If the trend of the data appears to be a straight line, insert the best line on the plot and include also the equation of the best line. From the equation, you can read off the slope and intercept. The acceleration can be calculated from the intercept b, since $b = \ln(a/2)$. The unit of a will be cm/s^2.

Show your calculation of a on the hand-in sheet.

Constant Acceleration Hand-in Sheet **Name:** ____________________

Partner: ___________________

What to turn in:

1. this sheet, completed
2. curved graph, followed by data table
3. linear graph, followed by data table
4. linear graph (logarithmic), followed by data table

Please staple together in this order.

Linear Analysis from graph #2

Slope from graph = ___________________

Show the equation for finding the acceleration from the slope, substitute the slope, and then give the acceleration with the correct units:

acceleration = ___________________

Logarithmic Analysis from graph #3

Intercept value from best-fit equation = ______________________

Using your logarithmic analysis, show that the intercept *b* equals an expression involving the acceleration. Solve for the acceleration.

acceleration = _________________

Finding the acceleration a third way:

Angle of inclined plane = _________________

Find $a = g \sin \theta$ = ____________________