Rotational inertia

Materials: *Blue mass kits, string, rotary motion sensor with stepped pulley, vernier caliper (to measure pulley radius), PASCO interface with appropriate software.*

1 Introduction

Newtons second law says the net force on an object is directly proportional to its acceleration, and the proportionality constant is the mass of the object. If the mass does not change as a function of time then

$$\vec{F}_{net} = \Sigma\vec{F} = m\vec{a} \tag{1}$$

The rotational version of this law says the net torque acting on an object about the rotation axis is directly proportional to the angular acceleration, and the proportionality constant is the moment of inertia, I. If the moment of inertia does not change as a function of time, the net torque is then expressed as

$$\vec{\tau}_{net} = \Sigma\vec{\tau} = I\vec{\alpha_{net}} \tag{2}$$

A complete discussion on this topic can be found in the textbook. In Eq. 2, the $\Sigma\vec{\tau}$ is the sum of all torques acting on the rotating system, I is the total moment of inertia for the rotating system and $\vec{\alpha}$ is the angular acceleration of the rotating part of the system.

From an experimental point-of-view, the moment of inertia of a system can be determined by measuring the net torque on a system and the angular acceleration and then plotting $\Sigma\tau$ vs. α. The slope of this plot will then be the moment of inertia for that system. This is very similar to the experiment earlier in the semester in which Newton's 2nd Law was investigated using a similar methodology. In order for this to work, however, both the net torque and the angular acceleration have to be measured and they need to be measured using separate techniques.

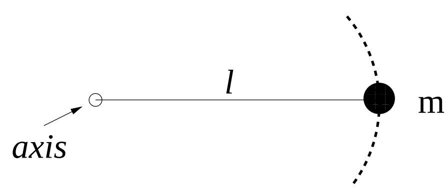

Figure 1: A single mass that can be treated as a point mass a distance l from the axis of rotation.

For a single point object rotating at distance l from an axis of rotation (Fig. 1), we have

$$I = ml^2. \tag{3}$$

If there are several point objects all rotating around the same axis, then just add these up, once for each object.

$$I = m_1l_1^2 + m_2l_2^2 + \cdots \tag{4}$$

We will use this to calculate the theoretical moment of inertia for the systems of two four point masses.

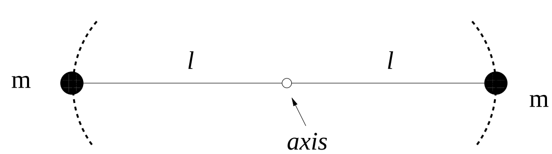

Figure 2: Two masses that can be treated as a two point mass system where each is a distance l from the axis of rotation.

For any rotating body with non-zero mass, there is an associated moment of inertia. The solid bar has mass that must be accounted for in this system. The moment of inertia for a continuous bar of total length w and mass m_{bar} being rotated about its center of mass can be calculated (see textbook for derivation) using

$$I_{bar} = \frac{1}{12} m_{bar} w^2 \tag{5}$$

The total moment of inertia for a rotating body can be calculated by summing all of the individual moments of inertia for each rotating body (as in equation 4 except also include the bar). That is

$$I_{total} = m_1 l_1^2 + m_2 l_2^2 + \frac{1}{12} m_{bar} w^2 \tag{6}$$

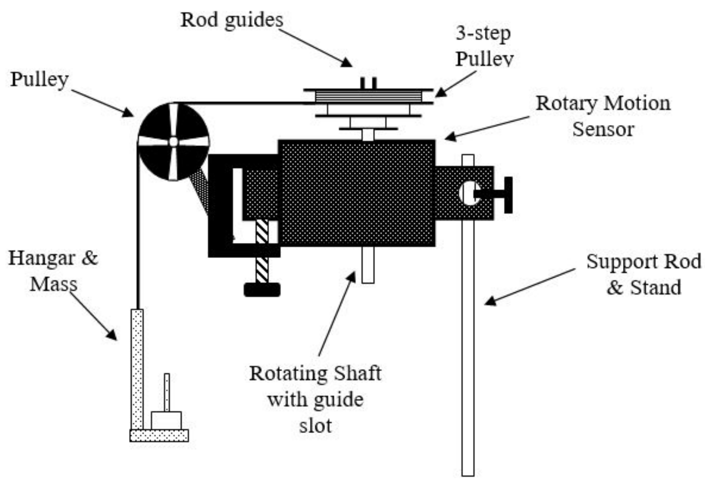

Figure 3: Side view of a rotary motion sensor similar to what is used for this experiment

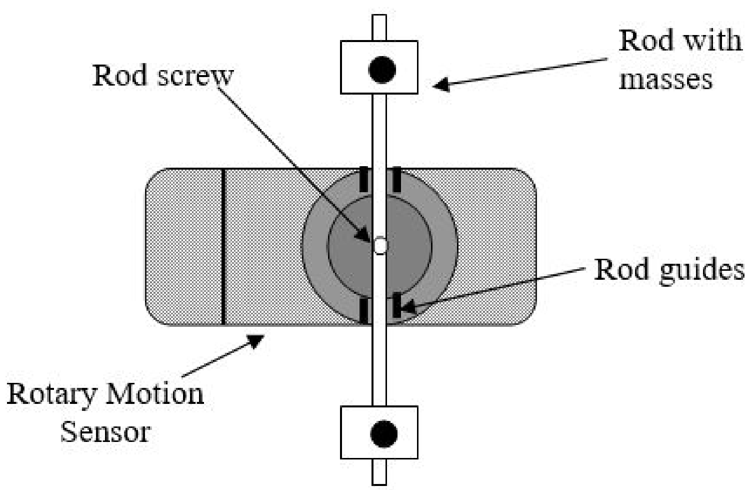

Figure 4: Top view of a rotary motion sensor similar to what is used for this experiment

Details of the data collection and analysis will be discussed in class and worked through in the response pages.

Responses and submission

Complete this section and attach (as required) the specified materials in order.

1. **Derive model equation**. Sketch the overall setup with a single bar and two masses symmetrically placed at the ends of the bar. Draw a free body diagram for the hanging mass. Draw the appropriate rotational free body diagram for the pulley noting that the tension in the string is the only applied force (torque) to the system. In a neat and well organized way, start with the basic principles of Newton's 2nd law and work through the translational and rotational force analyses to derive the model equation for this system, which is

$$\underbrace{R\left[m\left(g-a_{net}\right)\right]}_{\vec{\tau}_{net}} = I\vec{\alpha}_{net} \tag{7}$$

where 'R' is the lever arm for the string acting on the pulley, 'm' is the hanging mass, 'I' is the moment of inertia for the rotating system

2. Mass the bar and masses separately. Measure the overall length of the bar and the relevant radius of the pulley. Record each value here.

   Mass of bar ___________. Mass of individual masses ___________. Overall length of bar ___________.

- Attach the bar to the top of the pulley. Position each mass so that they are flush with the ends of the bar (i.e. 4; maximizing distance from the center).
- Configure the data collection software to plot angular position, angular velocity and angular acceleration (for the rotary sensor) as a function of time. (This may be provided as a pre-configured file, but, verify the settings are consistent with your setup).

3. **Data collection and reporting, setup 1**
   - Measure the angular acceleration for a chosen hanging mass. Repeat for enough masses to have a sufficient amount of data so that you can be confident in the measured relationships and values.
   - **Attach** a summary of your collected data in a neat, organized and properly labeled table.
   - **Attach** a plot of the net torque τ_{net} vs measured angular acceleration α_{net} (c.f. 7) that also includes an appropriate fit to extract the measured moment of inertia for the rotating system.

4. Show (or briefly explain) the steps to extract the measured moment of inertia from your data. Clearly indicate your measured value.

5. Specifically show the calculation for the predicted moment of inertia for the rotating bar and mass system.

6. Calculate the percent difference between your theoretical value and experimental value via

   $\%\ \text{Diff} = \frac{I_{ex} - I_{th}}{I_{th}} \times 100\% =$

7. **Data collection and reporting, setup 2**

   Move the masses to a position that is halfway between the end of the bar and axis of rotation

   Measure and report this distance here: ____________.

   - Measure the angular acceleration for a chosen hanging mass. Repeat for enough masses to have a sufficient amount of data so that you can be confident in the measured relationships and values.
   - **Attach** a summary of your collected data in a neat, organized and properly labeled table.
   - **Attach** a plot of the net torque τ_{net} vs measured angular acceleration α_{net} (c.f. 7) that also includes an appropriate fit to extract the measured moment of inertia for the rotating system.

8. Show (or briefly explain) the steps to extract the measured moment of inertia from your data. Clearly indicate your final measured value.

9. Specifically show the calculation for the predicted moment of inertia for the rotating bar and mass system.

10. Calculate the percent difference between your theoretical value and experimental value via

    $\%\ \text{Diff} = \frac{I_{ex}-I_{th}}{I_{th}} \times 100\% =$