Force and Acceleration: Newton's 2nd Law of Motion

Materials: *PASCO interface, two photo gate timers, computer with Capstone software, meter-stick, dynamics track and cart, blue mass sets, mass balance, two 100-gram extra masses*

1 Purpose

The goal of this experiment is to analyze and confirm the relationship of force and acceleration described by Newton's 2nd law of motion. The student will investigate how the acceleration of a system behaves when one of the two parameters (Force or Mass) is varied. Emphasis will be put upon the analysis of the data with linearization by logarithms. Data analysis will be performed using Excel.

2 Introduction

Newton's 2nd law of motion links the acceleration of an object to the force applied to it through the physical resistance. This resistance is called *inertia* and quantified by the mass (m) of the object. If the object's mass is constant, then this relationship is

$$\vec{a} = \frac{\vec{F}_{net}}{m} \tag{1}$$

where the acceleration ($\vec{a}$) and the net force ($\vec{F}_{net}$) are vector quantities.

To properly investigate this relationship, each group will perform two experiments which test the two possible physical relationships. The first experiment will test the linear relation between *net force* and *acceleration* by varying the force applied to a dynamics cart on a track and then measuring the resulting acceleration. Equation 1 represents this theoretical relationship, so the experimenter can plot *acceleration* versus *Net Force* and then determine the mass of the cart from a linear fit to the data.

The second experiment investigates the inverse relationship between the *acceleration* and *mass* of a cart where experimenters will vary the mass of the cart while keeping the applied force constant. Plotting *acceleration* versus *cart mass* will reveal a curved (non-linear) relationship which must be analyzed using logarithmic analysis. The non-linear curve stems from the inverse relationship of the two variables. As part of this exercise, you will use log analysis to show that Eq. 1 can be linearized and hence written as

$$Ln(a) = -1 \times Ln(m) + Ln(F). \tag{2}$$

3 Procedure

3.1 Equipment Setup

Assemble your cart and track to look similar to the setup illustrated in Figure 1. For any step appearing to have already been completed, it is important to verify that it is, indeed, configured correctly as your data depend on it.

1. Place the two photo gates as far apart as possible while maintaining two things:
   (i) The cart must start moving *before* it enters the first photo gate (right-hand gate in Fig. 1) and
   (ii) The flag must pass through the second photo gate (left-hand gate in Fig. 1) *before* the hanging mass reaches the floor.
   Once the photo gates are in place, do *not* move them for the rest of the experiment.

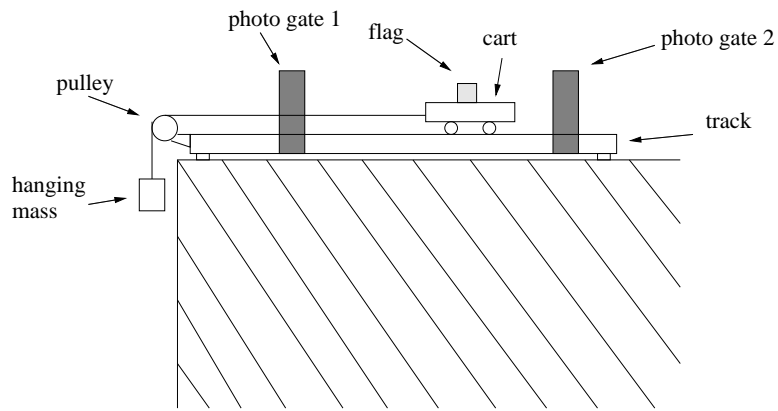2. Measure and record the distance between the two gates.

Figure 1: Setup for the Newton's 2nd Law experiment

3. Measure and record the mass of the cart without any extra masses.

4. Measure and record the width of the single black block on the 'picket fence' flag that will be mounted on top of your dynamics cart.

5. Attach the photo gates to the SW 750/850 interface and connect the interface to your computer. Turn on the interface.

6. If you have been provided with a file specific for this experiment, open that file and proceed to the next section. If not, follow these instructions to configure the sensors and associated options.

   (a) Open Capstone on your computer and choose any option. This can be changed later.

   (b) On the picture of the interface, click on the port one of the photogates is plugged into. Select the "photogate" from the list that appears. Do this for the second sensor as well.

   (c) On the left hand side are "tabs". Click on the one marked "Timer Setup".

   (d) Now select "Pre-Configured Timer", then make sure both photogates boxes are checked, and then select "Two photogates (Single Flag)" in the next section.

   (e) In section 4, make sure only "Speed in Gate 1" and "Speed in Gate 2" are checked. All others should be unchecked.

   (f) In section 5, type in the flag width and photogate spacing making sure you use the correct units.

   (g) In the window area where you have graphs and such, delete all those graphs and tables and such.

   (h) On the right-hand side is the data display options. Click and drag 2 instances (do it twice) the digits displace and drop them on the white area where data is displayed.

   (i) In each of those digits displays, click on the "select measurement" drop down menu and select "velocity in gate 1" for one window and "velocity in gate 2" for the other window.

   (j) Adjust the number of displayed digits on the "digit" windows so you are given 4 decimal places. This is done with a menu found on the "digit" window.

7. Navigate into the 'Timer Setup' options (left-hand side of the screen under 'tools') and in section 5 update the 'Flag Length' and 'Photogate Spacing' with the parameters from *your* setup (mind your units). Click 'save' (below section 6) and then click again on 'Timer Setup' to close that menu.

8. Click on the "record" button at the bottom to start data acquisition and run your cart (or hand) through the gates in successive order to test if the system is working. If not, verify all sensors are properly connected and then you may need to return to the sensor configuration instructions (above) to complete the setup.

3.2   Data Acquisition

3.2.1   Experiment 1: Varying the Applied Force

Each group will collect data on the acceleration of a cart on a track while being pulled by a constant force from a hanging mass. The mass hanging on the end of the string will be varied and the resulting change in acceleration determined from experimental data.

1. Place the empty cart on the track with the wheels in the grooves. Attach the string to the front of the cart.

2. Add sufficient mass to the blue hanger so that you have a total of 0.015-kg (*including* the hanger's mass ∼0.005-kg), attach it to the string and hang it over the pulley.

3. Begin collecting data via the Capstone software by clicking 'Record'

4. Release the cart from rest and *before* the flag passes through the first photogate. The cart must be moving before entering first photogate.

5. Stop the cart *after* it passes through the last photogate, *before* the hanger reaches the ground and record the velocity readings from each photo gate in a data table.

6. Repeat the measurements for total hanging masses of *20-g, 25-g, 30-g, 35-g, 40-g, 45-g* and *50-g*.

3.2.2   Experiment 2: Varying Cart Mass

Each group will collect data on the acceleration of a cart on a track while being pulled by a constant force from a hanging mass. The mass on the hanger will be held constant while the total mass of the cart will be varied by adding extra masses to the cart and the resulting acceleration determined from experimental data.

1. Place a *total* of 25-g on the end of the string suspended over the pulley (include the hanger's mass).

2. Release the cart as in the previous experiment and record the measured velocities.

3. Add 50-grams (0.050-kg) to the dynamics cart from the mass set. Repeat the measurement.

4. Repeat these measurements for *100-g, 150-g, 200-g, 250-g, 300-g, 350-g, 400-g, 450-g,* and *500-g* added to the cart.

5. Remember to measure the mass of the cart (should have already done this), carefully record the total mass added to the cart and both gate velocities for each run.

4   Analysis

4.1   Experiment 1: Varying the Applied Force

1. In Excel, generate a table of 'Hanging Mass', 'Gate 1 Velocity', 'Gate 2 Velocity', 'Applied Force', and 'Acceleration'.

2. Complete that data table with your measured values. You will need to calculate the applied force and acceleration (think kinematics) from measured parameters within your experimental setup.

3. Generate a plot of *Cart Acceleration* versus *Applied Force*. Perform a linear fit to these data. On the plot, include the 'equation of the fit line' and 'R-squared value'.

4.2　Experiment 2: Varying Cart Mass

1. Using the data from the second experiment, generate a data table in Excel with columns of the total mass of the cart, velocity through gate 1, velocity through gate 2, acceleration, natural log of total cart mass and natural log of the cart acceleration.

2. Generate a plot of *Natural Log of Acceleration* vs. *Natural Log of Cart Mass* (remember units and yes, they will be weird). Perform a linear fit to these data. On the plot, include the 'equation of the fit line' and 'R-squred value'.

5　Questions

Answer the following questions on clean and separate paper. They are to be included after the last plot.

1) Determine the experimental result for the mass of the cart by using the slope of the fit to your data in plot 1 [Hint: the relevant model is represented by Eq. 1.]

2) Compare your experimentally determined cart mass (from Q1) with the cart mass found by using a balance. Complete this comparison by way of a percent difference.

$$\% \ diff = \frac{experimental - actual}{actual} \times 100\% \tag{3}$$

3) Start with Eq. 1 and derive the [linearized] relationship (Eq. 2). This is the appropriate model to in analyzing the data in Experiment 2. Explicitly show what, in Eq. 2, corresponds to the *slope* and *y-intercept*.

4) Compare the experimentally determined slope from plot 2 to the theoretically predicted slope (determined in the previous question) for experiment 2. A percent difference calculation is appropriate.

5) Use the theoretical y-intercept from question 3 and the experimental y-intercept from graph 2 to determine the experimental value for the force F on the cart in the second experiment.

6) Compare this experimentally determined force with the *actual* weight of the hanging masses for experiment 2 via a percent difference.

7) *Derive an expression for the net acceleration ($\vec{a}_{net}$) of this system.*
Start with two free body diagrams – one for the cart with mass m_c and the second for the hanging mass (m_h) – then, continue with force analysis and Newton's 2nd law to show that

$$\vec{a}_{net} = \frac{m_h g}{m_h + m_c} \tag{4}$$

is indeed the proper expression for the net acceleration of the system.

6　References

- OpenStax, *Physics*, Chapter 4

- *Physics*, 8th edition, Cutnell and Johnson, Chapter 4.