Circular Motion and Centripetal Force

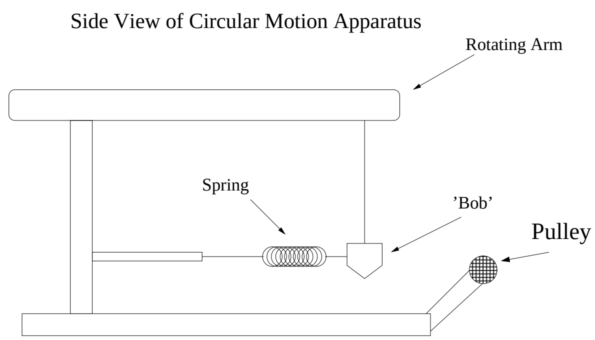
Materials: *Circular motion apparatus, stopwatches, laptop computers, short rulers, heavy mass sets with 50-g hangers*

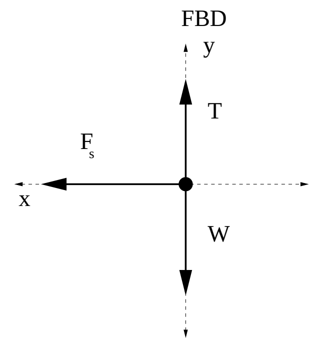
1 Purpose

The purpose behind this lab is for the student experimenters to investigate the centripetal motion and the force which keeps an object in circular motion. Students will continue to refine their analysis skills.

2 Introduction

The uniform circular motion of an object can be studied using this apparatus.





From the free body diagram (FBD) for this system, we see that the circular motion of the bob is maintained by the force supplied by the spring, F_s. This force is constant throughout the experiment. The sum of all forces (net force) in the x-direction is thus the spring force. When rotated at a constant rate, so that the bob hangs perfectly vertical, the bob undergoes uniform circular motion. The *net force required to maintain uniform circular motion* is, by definition, the centripetal force (F_c),

$$F_{net} = F_c = m\frac{v^2}{R} \tag{1}$$

where the linear velocity of the bob is v, its mass m and the radius of rotation R. Setting F_c equal to the sum of the forces in the $\hat{x}$-direction, we see

$$F_s = m\frac{v^2}{R}. \tag{2}$$

With this apparatus, the experimenter measures the rotational period, T, of the pendulum bob as a function of total bob mass, m, and so it is necessary to replace the v with an equivalence in terms of T. That is

$$v = \frac{2\pi R}{T}. \tag{3}$$

Then solving for T yields

$$T = \left(\frac{4\pi^2 R}{F_s}\right)^{1/2} m^{1/2}. \tag{4}$$

We can now measure the dependence of the the rotational period on the mass of bob. Note here that the theory in Eq. 4 predicts the relationship to be a power law form so logarithmic analysis is appropriate.

3 Procedure

Follow all of the instructions provided here and by your instructor. They will point out important aspects of the apparatus. Make sure you tighten all set screws and attachments after making changes.

3.1 Rotational Period Measurements

1. Carefully detach the pendulum bob and measure it's mass. Reattach it to the apparatus **without** attaching the spring.
2. Measure the radius of rotation for the pendulum bob when it is in its fully vertical position. (Do not rotate the system without the spring connected!)
3. Reattach the spring to the pendulum bob.
4. Rotate the system until the bob is perfectly vertical (above the point that you measured in the previous step) while being rotated. This requires a bit of practice.
5. Once you are comfortable with the apparatus and your ability to maintain a sufficient angular speed to keep the bob above the measured R, measure the time for **10** complete rotations of the bob and then calculate and record the time for *one* complete revolution (rotational period, T) in a table of the bob's *total mass* and *rotation period*. (Think carefully about what the period is and its relationship to the time you measured. They are not equal!)
6. Repeat the measurement two more times and average the three results. This will help to minimize uncertainties associated with starting and stopping the stopwatch.
7. Add 50-grams to the bob and repeat steps 4 through 6. Do this for 100-g, 150-g, 200-g, 250-g, and 300-g added to the bob. Do not exceed a safe load limit of the apparatus – this means you may not be able to add all 300-g to the bob if you cannot safely secure the masses to the top of the bob.

3.2 Spring Force Measurement

Measure the amount of force the spring exerts on the bob when the bob is vertical (hanging above the measured R) by attaching a string to the opposite side of the bob and suspending mass over the small pulley on the apparatus.

1. Attach a string to the bob. Hang the string over the small pulley.
2. Place a hanger on the end of the string. Add mass until the bob is vertical (at R) while not rotating.
3. Record the amount of mass suspended on the end of the string (m_h). This is the mass necessary to 'balance' the spring force. Thus, we now have a direct measurement of the spring force since, while hanging statically, the spring-'hanging mass' system is in equilibrium. That is

$$F_{net,(3.2)} = 0 = \vec{F}_S - m_h g \rightarrow F_S = m_h g. \tag{5}$$

4 Analysis

1. *Data Table*: Generate a table with columns for the total mass of bob, $m_{total} = m_{bob}$+added mass; each period measurement, T; average of the three period measurements, T_{avg}; square root of total bob mass, $\sqrt{m_{total}}$; natural log of the period, $\ln[T_{avg}]$; and natural log of total bob mass, $\ln[m_{total}]$. (Use SI units for all.)

2. Under your data table, explicitly show the calculation for F_s directly measured (by way of the masses) in section 3.2.

3. *Plot 1*: Plot the *natural log of period* vs. *natural log of total bob mass* and fit the result with a straight line.

4. *Plot 2*: Plot the *period* vs. *square root of total bob mass* and fit the result with a straight line.

5 Questions

Answer the following question clearly and concisely.

1. Start with Equation 4 and show the steps to arrive at

$$\ln[T] = \frac{1}{2}\ln[m] + \ln\left[\left(\frac{4\pi^2 R}{F_s}\right)^{1/2}\right]. \quad (6)$$

   which is the model that applies to the data plotted in Plot 1 (*Natural log of period* vs. *natural log of total bob mass*). What is the physical significance of the slope of of the first graph you generated?

2. Using a percent difference comparison, comment on how well the theoretical and experimental slopes match for Plot 1. Recall that

$$\%diff = \frac{measured - theory}{theory} \times 100 \quad (7)$$

3. Use the y-intercept from Eq. 6 and the numerical y-intercept from the fit line on Plot 1 to solve for the force supplied by the spring to the rotating system, F_s. This is your 'measured' F_S. (You will also need to use the measured value of R.)

4. With a percent difference calculation, compare the spring force (F_S) you calculated in question 3 with the force found from the section 3.2 by direct measurement. Here the value from Question 3 is the 'measured' value and the value found directly in Section 3.2 is the 'theory' value.

5. Use your slope from Plot 2 (T_{avg} vs $\sqrt{m_{total}}$) and Eq. 4 to determine the spring force exerted on the pendulum bob while in motion. This will require you to compare Eq. 4 to the linear fit from Excel.

6. How does this value of F_s compare to the value you determined in Section 3.2? Use the percent difference comparison equation (Eq. 7). Here you will use the value from Question 5 as the 'measured' value and the value directly determined in Section 3.2 as the "theory" value.

6 References

- OpenStax *Physics*, Chapters 5 and 6
- *Physics*, 8^{th} edition by Cutnell and Johnson, Chapter 5