Rotational Inertia

Materials: *Rotodynes mounted on ceiling rails, stop watches, blue mass sets, 2-meter-sticks*

1 Purpose

The goal of this laboratory is to experimentally determine the moment of inertia for a rotodyne wheel, rotodyne with 2 point masses and a rotodyne with 4 point masses. With this data, the student will verify the point mass formula for moment of inertia as applied to 2-point mass and 4-point mass systems.

2 Introduction

2.1 Moment of Inertia Theory

Newtons second law says the net force on an object is directly proportional to its acceleration, and the proportionality constant is the mass of the object. If the mass does not change as a function of time then

$$\vec{F}_{net} = \Sigma\vec{F} = m\vec{a} \tag{1}$$

The rotational version of this law says the net torque acting on an object about the rotation axis is directly proportional to the angular acceleration, and the proportionality constant is the moment of inertia, I. If the moment of inertia does not change as a function of time, the net torque is then expressed as

$$\vec{\tau}_{net} = \Sigma\vec{\tau} = I\vec{\alpha_{net}} \tag{2}$$

A complete discussion on this topic can be found in the textbook. In Eq. 2, the $\Sigma\vec{\tau}$ is the sum of all torques acting on the rotodyne, I is the total moment of inertia for the rotodyne and $\vec{\alpha}$ is the angular acceleration of the rotodyne.

From an experimental point-of-view, the moment of inertia of a system can be determined by measuring the net torque on a system and the angular acceleration and then plotting $\Sigma\tau$ vs. α. The slope of this plot will then be the moment of inertia for that system. This is very similar to the experiment earlier in the semester in which Newton's 2nd Law was investigated using a similar methodology. In order for this to work, however, both the net torque and the angular acceleration have to be measured and they need to be measured using separate techniques.

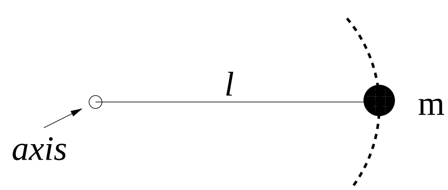

Figure 1: A single mass that can be treated as a point mass a distance l from the axis of rotation.

Before the specifics of the data collection are covered, it is necessary to discuss the background for the moment of inertia theory. For a single point object rotating at distance l from an axis of rotation (Fig. 1), we have

$$I = ml^2. \tag{3}$$

If there are several point objects all rotating around the same axis, then just add these up, once for each object.

$$I = m_1 l_1^2 + m_2 l_2^2 + \cdots \tag{4}$$

We will use this to calculate the theoretical moment of inertia for the systems of two and four point masses.

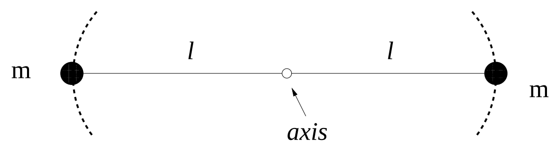

Figure 2: Two masses that can be treated as a two point mass system where each is a distance l from the axis of rotation.

In the case that the distance the masses are from the axis of rotation is the same ($l_1 = l_2 = \cdots$) and the masses are all the same (see Fig. 2; $m_1 = m_2 = \cdots$), then Eq. 4 reduces to

$$I_{2PTMass} = 2ml^2. \tag{5}$$

Having four equal point masses would then reduce Eq. 4 to

$$I_{4PTMass} = 4ml^2. \tag{6}$$

2.2 Angular Acceleration and Net Torque

Finding the moment of inertia of the system requires that the angular acceleration be determined. This is done by dropping a mass from a given height while attached to the rotodyne (Fig. 3). Dropping the mass from rest from a known height and timing the fall will allow for the acceleration to be calculated.

$$a = \frac{2h}{t^2} \tag{7}$$

Since the mass is attached to the string the rotodyne must also rotate at the same tangential acceleration as the falling mass. The angular acceleration can be found by the equation

$$\alpha = \frac{a_{tan}}{R} \tag{8}$$

where R is the radius to where the string is attached.

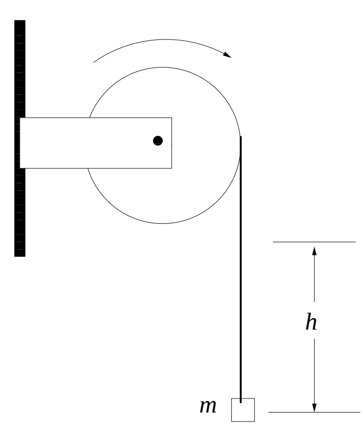

Figure 3: Sketch of the rotodyne system without attached masses.

The net torque is determined by analyzing the FBD for Figure 3 and computing the torque using $\tau = RF\sin(\theta)$. Note that in this setup, the string is always perpendicular to the edge of the wheel, so $\theta = 90°$ and $\sin(90°) = 1$. Analysis of the FBD yields a force of

$$F = m(g - a). \tag{9}$$

Combining this with R yields a net torque of

$$\Sigma\tau = Rm(g - a). \tag{10}$$

The net torque and the angular acceleration are now determined experimentally and separately and a plot of $\Sigma\tau$ versus α can be made. The slope of that plot is the moment of inertia.

2.3 Composite Systems

The moment of inertia for a composite system is found by taking the moment of inertia's for each component and adding them together as long as they share a common axis of rotation. For this experiment, the moment of inertia for the rotodyne cannot be determined theoretically given the complexities of its geometry, however, it can be determined experimentally. Once its moment of inertia is known, the moments of inertia for composite systems such as point masses mounted on the rotodyne can be determined.

For example, the moment of inertia for the rotodyne with 2 point masses attached can be determined experimentally and is one of the systems you will measure. That measurement represents the total moment of inertia which is the sum of the two contributing parts.

$$I_{total} = I_{rotodyne} + I_{2\ pt\ masses\ alone} \tag{11}$$

Armed with this knowledge and your measured values, you can extract out an experimental value for the moment of inertia for the 2 point masses. You can then compare this with your predicted results from the point mass formula.

3 Procedure

This is the first draft of this handout. Directions are Spartacus.

- Measure the mass of each "point mass" and average the results for your m value.
- Measure the distance from the axis of rotation (center of axle) to the point at which the masses are attached. It is the farthest point out from the axle unless you have been told otherwise. This is your l value. (You may find it easier to measure the effective diameter – total distance between mass mounting points spanning the center – and then divide that number by 2).
- Measure the distance from the axis of rotation to the point at which the string touches the inside of the rim of your rotodyne. This is the distance R for your net torque and angular acceleration calculations. (You may find it easier to measure the diameter and divide that by two instead of guessing where the center is).
- Pick a convenient height from which to drop the hanging mass and use the 2-meter stick to line it up every time (marking the string with a small piece of tape for consistency can also be helpful). Use the provided pad to break the fall of the hanging mass. A release point near eye level is very useful. Also, a longer distance will help reduce the fractional uncertainty relating to the stop watch operator's reaction time.
- With no point masses on the rotodyne, place a total of 50 grams on the string (5 grams for the hanger and 45 grams more) and release the mass from rest timing the fall of the mass to the floor.

- Repeat the time measurement two more times and calculate the average. This is to limit start/stop uncertainties and to act as a check for minor mistakes.
- Repeat the measurements for 75, 100, 125, and 150 grams on the string.
- Attach to of the point masses to the outer most attachment points directly opposite from each other in order to maintain balance.
- Repeat the measurements as you did for the rotodyne alone system.
- Attach the remaining two masses at the positions that are rotated 90° to the other first two masses. Repeat the measurements.

4 Analysis

Outlined here are general steps to complete the analysis. Use these, along with directions from your instructor, to complete the experiment.

- Generate a table for each setup (3 total) that has the hanging mass, average time, acceleration, angular acceleration, and net torque. EXCEL is your friend – if you do not know how to use formulas, this would be an excellent time to ask so that you may learn.
- Generate a plot of $\Sigma\tau$ versus α for each data set. Make sure you use correct labels, significant figures, and other necessities.
- Determine the experimental value for the moment of inertia for a 2 point mass system and a 4 point mass system from the three measured moments of inertia from the plots.
- From the measured values of m and l, calculate the theoretical values for the moments of inertia for these two systems using Eqs. 5 and 6.
- For both the 2-added mass and 4-added mass systems, compare the experimental results to the theoretical prediction with a percent difference: $\frac{I_{exp}-I_{theory}}{I_{theory}} \times 100\%$.

5 Questions

There are no additional questions prescribed by this handout. Be sure to completely address any other questions or parts that may be prescribed by your instructor.

References

- See e.g.: OpenStax College, *College Physics*. 21 June 2012 <http://cnx.org/content/col11406/latest>