Simple Harmonic Motion

Materials: *Simple pendulum suspended from the ceiling rail, bench mounted spring-mass oscillators, stop watches, mass hanger, mass sets, 2[m] meterstick*

1 Purpose

The goal of this laboratory is to investigate simple harmonic motion with two different systems and determine the significant constants of the individual systems. Students will use their graphing skills to analyze the data which includes linear and logarithmic analysis.

2 Introduction

'Periodic motion' describes a situation where an object regularly returns to a particular position after a fixed time interval. Simple harmonic motion is a special kind of periodic motion

i. that involves a restoring force that is dependent on displacement from an equilibrium position and has a direction that is always opposite that of the displacement from equilibrium

ii. whose frequency (or period) is independent of amplitude

iii. whose amplitude is determined entirely by how the oscillator is set into motion

Examples of simple harmonic oscillators are simple pendulums (a mass on the end of a length of string), physical pendulums (mass at the end of a long metal rod), mass-spring systems which oscillate along the spring axis, and atoms within the structure of molecules. In this laboratory the experimenter will investigate two systems, the mass-spring system and the simple pendulum, and through their analysis skills determine constants specific to the system.

The mass-spring system consists of a mass attached to the end of a spring and suspended in air on a lab bench. When the mass is displaced from its resting position by pulling on the mass and then releasing the mass begins to oscillate back and forth. The oscillation follows a simple harmonic path and can be modeled with the mathematical equations of simple harmonic motion. For the mass-spring system, the simple harmonic motion follows the relationship of

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{1}$$

where T is the period of oscillation, m is the mass attached to the spring, and k is the stiffness constant of the spring.

The simple pendulum system consist of a mass attached to the end of a string and suspended from the ceiling rails. The length of the string suspending the mass dictates the period of the oscillation of the pendulum. The formula is

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{2}$$

where L is the length of the string from the connection point to the center of the mass and g is the gravitational acceleration at the surface of the planet which in this case is the earth.

3 Procedure

The experimenter will perform two experiments to measure the stiffness constant of a spring, k, and a third to measure the gravitational acceleration for an object near the Earth's surface, g.

3.1 Measurement of the Spring Constant by the Force Method

1. Measure the distance from the floor to the bottom of the spring *without* the mass hanger. Record this as x_0, the equilibrium position of the spring.

2. Attach the 50-g hanger to your spring system. Add another 50-g to the hanger to make it an even 100-g.

3. Measure the distance from the floor to the bottom of the spring (x_i). Record this distance and the total amount of mass hanging on the spring (m_i).

4. Repeat the measurement for 200-g, 300-g, 400-g, 500-g, and 600-g total hanging mass (remember this includes the mass of the hanger as well).

3.2 Measurement of the Spring Constant by the Oscillation Method

1. Attach 100-g to the end of the spring.

2. Set the system into motion (vertical oscillation) by gently lifting up on the bottom of spring and releasing.

3. Measure the time (t) it takes to complete **10 oscillations**. Repeat this measurement two more times and average the results ($\bar{t}$). Look for anomalies in the collected data and repeat if necessary.

4. Record the value for the period of oscillation (T) in a data table along with the mass of the system. **NOTE: The period of oscillation is the total measured time divided by 10 oscillations** ($\bar{t}$/[# Oscillations]).

5. Repeat this for 5 additional masses (such as 150-g, 215-g, 320-g, 475-g, and 700-g). Record the total time, period of oscillations and the actual mass values in the data table.

3.3 Measurement of Earth's Gravitational Acceleration by the Oscillation Method

1. Set the length of string of the pendulum to 0.70-m (something within ± 3 [cm] is okay, but be sure to record the **actual** length of string from the attachment point to the center of the bob.

2. Set the pendulum into **small** oscillations and measure the time (t) for 10 complete oscillations of the pendulum. Repeat the measurement two more times and average the results ($\bar{t}$).

3. Record the actual length of the pendulum (L) and the period of oscillation of the pendulum (T). Remember, the period is one complete oscillation ($\bar{t}$/[# Oscillations]).

4. Repeat the measurement for 0.60-m, 0.50-m, 0.40-m, 0.30-m, and 0.20-m and record the results.

4 Analysis

4.1 Analysis of the Spring Constant by the Force Method

1. Generate a data table with columns for the mass hanging from the spring (m_i), the force applied by the hanging mass, the distance the bottom of the spring is from the floor (x_i) and the stretch length of the spring ($\Delta x_s = x_i - x_0$; note that this is actually a displacement so the sign should *not* be ignored). Make sure all measurements are in standard SI units.

2. Generate a plot of *Applied force* vs. *Distance from equilibrium* and fit the plot with a straight-line.

4.2 Analysis of the Spring Constant by the Oscillation Method

1. Generate a data table with a column with the hanging mass, the period of oscillation, square root of the mass, the natural log of the mass, and the natural log of the period of oscillation.

2. Generate a plot of *Natural Log of period* vs. *Natural log of hanging mass* and fit the plot with a linear fit.

3. Generate a plot of the *Period of oscillation* vs. *Square root of total mass*. Fit the plot with a linear fit.

4.3 Analysis of Earth's Gravitational Acceleration by the Oscillation Method

1. Generate a data table with columns for the length of the pendulum, the period of oscillation, and the square of the period of the oscillation.

2. Generate a plot of the *Period of oscillation* vs. *Square root of pendulum length* and perform a linear fit.

5 Questions

Address all parts of the analysis sections, these questions in addition to anything else prescribed by your instructor.

5.1 Questions for the Spring Constant by the Force Method

From Hooke's law , the applied force is directly related to the displacement of the spring via the stiffness constant of the spring. The algebraic relationship is

$$\vec{F}_{applied} = -k\Delta\vec{x}_s \tag{3}$$

where k is the stiffness constant, and $\Delta\vec{x}_s$ is the displacement from equilibrium (stretch length).

1. Using your plot from the first experiment, determine the stiffness constant for the spring. Show your work.

5.2 Questions for the Spring Constant by the Oscillation Method

1. In the plot of *Period* vs. *Square root of mass*, what is the value (with units) of the slope?
2. Using the theoretical Eq. 1 and information from your plot, determine the value of k (stiffness constant). Show your work. (Hint: Compare the slope from the theoretical expression to the slope on your plot).
3. From the *Natural Log Period* vs. *Natural Log Mass* plot, what is the slope of the line? What does theory say it should be? (Hint: To understand the theoretical result, you need to take the natural log of both sides of Eq. 1 and manipulate the equation into a useful form.)
4. You have determined the spring constant k using two different methods. Compare your determined k from section 5.1 to that of section 5.2 using a percent difference, i.e.:

$$\%\text{ difference} = \frac{k_1 - k_2}{0.5 \times (k_1 + k_2)} \times 100\% \tag{4}$$

5. Which method of determining the spring constant k do you think is more accurate? Support your claim.

5.3 Questions for Earth's Gravitational Acceleration by the Oscillation Method

1. Determine a value for g, earth's gravitational acceleration. Show your work. (Hint: Use the Eq. 2 to help in your understanding of the plotted data and extract the value of g.
2. How does this value compare with the accepted value of g? Use

$$\%\text{ error} = \frac{g_{meas} - g_{theory}}{g_{theory}} \times 100\% \tag{5}$$

and use the accepted value of 9.80-m/s^2 as the theory value.

References

- *Physics* 8^{th} Edition, Cutnell and Johnson,Chapter 10.
- OpenStax College, *College Physics*, 21 June 2012 <http://cnx.org/content/col11406/latest>.