Constant acceleration 2

Materials: Dynamics track with stand (to control incline), dynamics cart, photogate and appropriate hardware interface

1 Introduction

This exercise will examine the motion of an object moving with constant acceleration in one dimension. Measuring the velocity of a low-friction cart traveling through a series of set displacements on an inclined plane provides enough information to determine the average acceleration of the cart. Assuming the acceleration is constant (or equivalently that only the average acceleration is desired), the time-independent kinematic equation

$$v_2^2 = v_1^2 + 2a\Delta x \tag{1}$$

is valid and relates the cart's displacement Δx, initial velocity v_1, final velocity v_2 and acceleration a without relying on a time.

In this lab, work with the special case where the cart is released from rest so that $v_1 = 0$ so that equation 1 becomes

$$\rightarrow v_2^2 = \cancelto{0}{v_1^2} + 2a\Delta x \rightarrow \Delta x = \frac{1}{2a} v_2^2 + 0 \tag{2}$$

2 Experiment

The main apparatus for this experiment is shown in Fig 1. The inclination angle θ can be set between 4 and 10 degrees. **Procedure**: In general, release the cart from rest with the leading edge of the cart at the same position (e.g 20 cm) and place the photogate at several different points along the track to determine the speed of the cart traveling through the gate at each measured displacement.

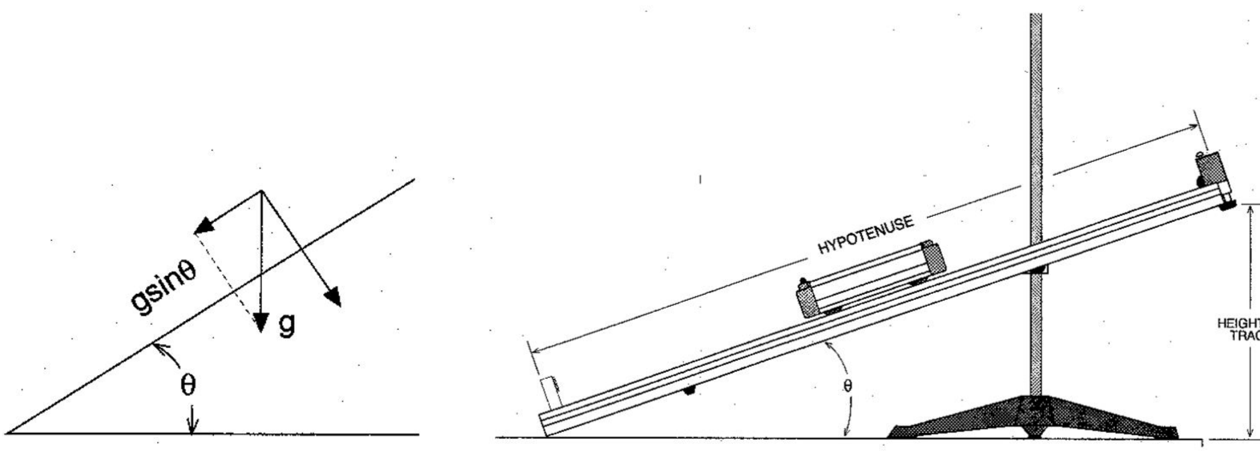

Figure 1: Example of (right) the inclined dynamics track for constant acceleration measurements and (left) a vector component breakdown of the acceleration vector for the cart. Note that θ here is greatly exaggerated and in practice should be on the order of a few degrees.

2.1 Application notes

Before starting the experiment, read and consider the following points.

- The velocity of the cart is determined by measuring the width of the flag $w = 0.025$ [m] passing through the photogate. The software records the time during which the photogate is blocked Δt and so the resulting average velocity through the photogate is determined by $\frac{w}{\Delta t}$
- Practice. Before collecting data, do a few practice runs to get a feel for how the equipment will operate and any potential difficulties to work out before beginning data collection.

- The cart is not a point object. Use the front of the flag on the cart as a constant reference point. Here, the bottom of the track is a bumper that will keep the cart from going off of the track. This bumper will stop the cart and push it back up the track.
- For each distance, take at three timed runs and use an average of these three for the analysis.
- Use the full length of the track and take reasonable step sizes to collect a good quality data set. Beginning with a 1.80 m distance and working towards a 0.60 m distance in steps of 0.10 m provides enough data for this experiment and the following analysis. If there is an obvious issue with a single run (e.g. cart falls off the track, time was not started or stopped close enough to the cart's actual start and stop) simply disregard and repeat the measurement. However, one should not disregard a measurement without a good reason; 'that time does not seem right' is not a good reason.
- Record the raw data in a simple table and then use a program to perform any repeated calculations. For example, write a formula in Microsoft Excel to calculate the average time for each run based on the three inputted raw times and then use that calculated average time to determine the velocity through the gate. A good rule of thumb is that any calculation done more than twice when processing your data can be automated to be more time efficient.

2.2 Analysis

Visualize and analyze the data with graphs as discussed in §2.2.1 – §2.2.2

2.2.1 Graph 1, visualize the raw data

The first step in analyzing any data is to get a sense for what it looks like. Formalize a data table in the format of Table 1. Make a plot of the displacement Δx as a function of velocity v_2 (think carefully about what this should look like based on the model). Since this plot is not linear and there are discrete data points, there should be no fit line and no line connecting each data point. Be sure to include the required elements such as an informative title, your name, properly labeled axes, etc.

Table 1: Table format for raw data.

Initial position x_i (m)	Final position x_f (m)	Distance Δx (m)	Time t_1 (s)	Time t_2	Time t_3 (s)	Average Time $\bar{t}$ (s)	Velocity $v = \frac{w}{t}$ (m/s)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

2.2.2 Linear analysis

One way to extract the cart's acceleration from this non-linear data is to come up with a way to plot it that yields a straight line. Starting with the model equation (eqn 2) for the special case applicable to this experiment (i.e. initial velocity v_1 is zero) one can see that replacing v_2^2 with a linear parameter (i.e. $u \equiv v_2^2$) results in a standard linear expression. That is

$$\begin{aligned}\Delta x &= \left(\frac{1}{2a}\right) v_2^2 + 0 \\ &= \left(\frac{1}{2a}\right) u + 0 \quad | \, u \equiv v_2^2 \\ &\updownarrow \\ Y &= (\text{slope})X + (\text{y-int})\end{aligned} \tag{3}$$

From this model equation, the slope of a linear fit line is '(slope) $= 1/(2a)$' and the y-intercept is zero (in the model).

Make a new data table like Table 2, plot displacement as a function of v_2^2 and perform a linear fit. In a program such as Microsoft Excel, when using the 'trendline' tool, make sure to include the equation on the plot and update the equation text to include the appropriate variables and units.

Table 2: Table format for linear analysis.

Displacement, Δx (m)	Velocity, v_2 (m/s)	v_2^2 [$(\text{m/s})^2$]
$\vdots$	$\vdots$	$\vdots$

To submit

In this order in a single PDF, assemble and submit

1. Completed ‘submission pages’
2. Non-linear graph of raw data (Δx vs v_2) followed by data table (§2.2.1)
3. Linear graph (Δx vs v_2^2) followed by data table (§2.2.2)

Submission page

2.3 Linear analysis from graph 2 (§2.2.2)

Slope from fit equation on graph 2 (Δx vs v_2^2) = ____________________

Write the equation for determining the acceleration from the slope, substitute the slope from your fit line and calculate the acceleration.

Acceleration of cart = ____________________

2.4 Predicted acceleration

Angle of the inclined plane = ____________________. Calculate $a = g \sin(\theta) =$ ____________________.

2.5 Experimental uncertainty

2.5.1 Estimating the uncertainty

The experimental uncertainty for this experiment includes measurements of time and distance. Use the spaces and prompts below to complete the uncertainty estimate for each measurement and then the overall experiment.

- Flag width, w

  Estimated uncertainty, $\delta w =$ __________ ; Measured $w =$ __________ ; Fractional value $\frac{\delta w}{w} \times 100\% =$ __________

- Time flag blocks photogate, measured Δt

  (Note: For this estimate, use the sample rate of the photodetector consider the set sample rate of the sensor – this roughly corresponds to the shortest possible event that can be measured.)

  Sample rate = __________ ; Estimated uncertainty, $\delta t \approx \frac{1}{\text{sample rate}} =$ __________ ;

  Shortest measured $t =$ __________ ; Fractional value $\frac{\delta t}{t} \times 100\% =$ __________

- Distance cart measured down the track, measured Δx

  Est. uncert., $\delta \Delta x =$ __________ ; Smallest meas $\Delta x =$ __________ ; Fractional value $\frac{\delta \Delta x}{x} \times 100\% =$ __________

- Inclination angle of the track, θ

  Estimated uncertainty, $\delta\theta =$ __________ ; Measured $\theta =$ __________ ; Fractional value $\frac{\delta\theta}{\theta} \times 100\% =$ __________

- **Overall**: Combine the individual estimated fractional uncertainties to determine an overall estimated uncertainty for this experiment, $\delta(\text{all})$

  $\delta(\text{all}) = \sqrt{\left(\frac{\delta w}{w} + \frac{\delta \Delta t}{t}\right)^2 + \left(\frac{\delta \Delta x}{\Delta x}\right)^2 + \left(\frac{\delta\theta}{\theta}\right)^2} =$

2.5.2 Comparing the result

Compute the percent difference between the predicted acceleration and the result from each graph with an appropriate trendline. *Briefly* comment on your result and whether or not the result falls within a reasonable range of uncertainty.

- Acceleration from graph 2

  $\%\text{Diff} = \frac{a_g - a_{\text{predict}}}{\frac{1}{2}(a_g + a_{\text{predict}})} \times 100\% =$