Moment of inertia of a rotational system

1 Introduction

1.1 Purpose

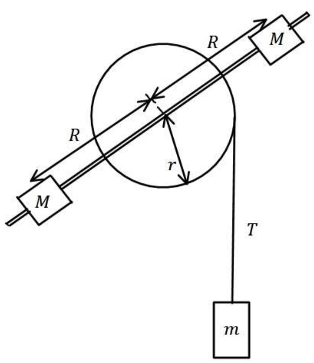
The goal of this lab is to test the rotational version of Newton's 2nd law. This will be achieved by varying the moment of inertia for a system and measuring the rotational behavior in response to a linearly applied force.

1.2 Theory

Newton's second law as applied to a rotating system states that the net torque $\sum \tau$ for a system is directly proportional to the angular acceleration α of the system about the axis of rotation (assuming the mass distribution remains constant). The proportionality constant is effectively a 'mass' term for rotation that is called the *moment of inertia*, I. That is

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = I\vec{\alpha} \tag{1}$$

1.3 Testing the hypothesis



We will test this law using data collected with a system consisting of a rotating rod attached to a hanging mass through a string over a pulley. Fig **??** showsthat there are two masses M that can be placed at various distances R from the center of rotation and that there is a hanging mass m attached to a pulley with radius r fixed to the rotating bar. The distance r is the lever arm used to calculate the torque applied to the rotating apparatus by the tension T in the string (*NOTE: the upper- and lower-case variables refer to different physical entities*)

To test Newton's 2nd law for rotation, the following model can be developed for this system

$$\frac{g}{\alpha} = \left(\frac{2M}{mr}\right) R^2 + \frac{I_0}{mr} + r \tag{2}$$

The instructor will work through the derivation of eqn 2. Write out the derivation for yourself so that you understand the origin of the model.

1. Apply Newton's 2nd law to the hanging mass
2. Apply Newton's 2nd law for rotation to the rotating part of the system, with total moment of inertia I
3. Combine these force analyses appropriately to eliminate tension T from the equation of motion
4. Substitute $I = I_0 + 2MR^2$ for the moment of inertia, where MR^2 is the contribution from each of the two masses on the rod, and I_0 is the moment of inertia of everything else that is rotating, inculding the rod and the metal axle that passes through the measuring unit
5. Rearrange to end with eqn 2

1.4 Visualization

Equation 2 shows that, if the theory is correct, a graph of g/α as a function of R^2 should be a straight line with

$$\text{Slope} = \frac{2M}{mr} \quad \text{and} \quad \text{intercept} = r + \frac{I_0}{mr} \tag{3}$$

2 Experiment

1. Choose several distances R for he masses M on the rod and measure the angular acceleration α for each.

   Note: The hanging mass m must remain constant

2. Use units of centimeters for the distances and use units of grams for masses. then the moment of inertia will have units of 'gram·(cm^2)'

3. The instructor will discuss how to determine the angular acceleration by using the Pasco software to find the slope of the *angular velocity* versus *time* plot.

Table: Data for rotational apparatus

$2R$ [cm] guide	$2R$ [cm] actual	α (rad/s^2)	R^2[cm^2]	$\frac{g}{\alpha}$[cm]
36				
33				
30				
26				
22				
16				
7.5				

Additional information

Hanging mass, m	25	[grams]
Pulley diameter, $2r$	2.90	[cm]
Rod length, l	38.0	[cm]
Rod mass	25.6	[grams]

Notes for the table

1. The first column is a guide for the distance between the two orbiting masses. When they are positioned to balance the rod, measure the actual distance between the centers of the two masses and enter that in the second column.

2. to measure $2R$, measure from the outside edge of one mass to the inside edge of the other (this is effectively averaging the separation distance of the two orbiting masses)

3. The angular acceleration in the second column is determined using the Pasco software (linear fit to the angular velocity data)

4. Calculate g/α using 981 cm/s^2 and the third column

References

e.g. see [OpenStax, *Physics*, Chapter 8] or [*Physics*, Serway and Jewett 10th ed]

Responses and submission

Complete this section and attach (as required) the specified materials in order.

1. Excel (data) table with proper formatting (e.g. clear labels, neatly organized, etc)
2. Graph (Excel) of g/α as a function of R^2. Make sure the trend line and correlation coefficient are shown. Make sure the slope and intercept have at least three significant digits.
3. Calulation of the mass or each 'orbiting mass'. Use the slope expression and the slope value from your graph. Set them equal to each other and then solve for M. The result should be in the 50 [g] to 100[g] range.

   Show your work here.

4. Calculation of the experimental value for I_0, the moment of inertia of the rod, pulley and the metal axle of the rotation device. Use the intercept expression and the intercept value from the trendline. Set them equal and solve for I_0.

   Show your work here.

5. Calculate the theoretical value of the moment of inertia of the rod alone, using the formula $I_{rod} = \frac{1}{12} m_{rod} l^2$. Based on your answer, what do you estimate is the moment of inertia of the pulley and the axle of the rotary sensor?

   Show your work here