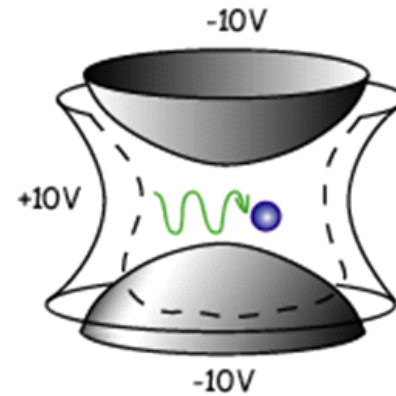
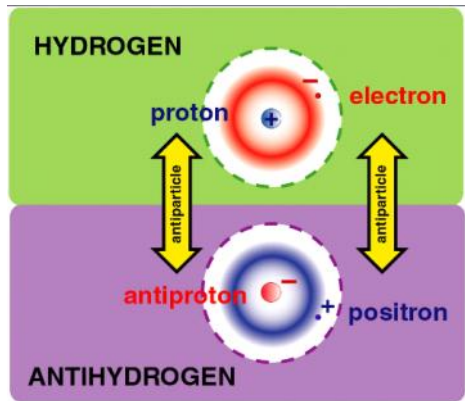


Low-Energy Precision Studies in Quantum Systems



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IUCSS Summer School on the Lorentz- and CPT-violating Standard-Model Extension
Indiana University, Bloomington, June 3-9, 2012

Outline

Penning Trap

hep-ph/9809543
(PRD paper)

- what is it?
- what are the Lorentz-violating signals?
- experimental results

Antihydrogen

hep-ph/9810269
(PRL paper)

- basics
- what are the Lorentz-violating signals?

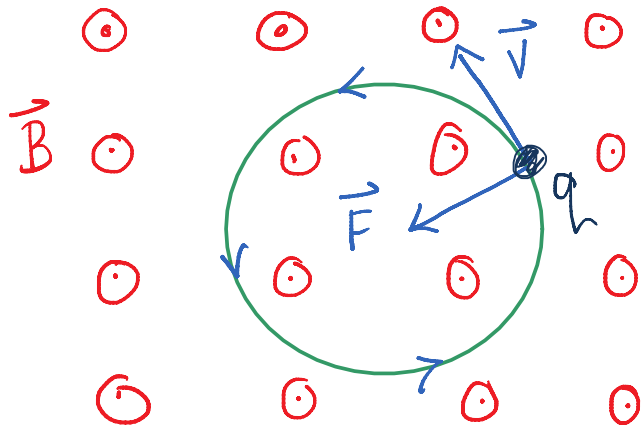
Penning-trap physics

The Penning trap is a device that confines charged particles.

Uses include high-precision measurements of particle properties

- charge-to-mass ratio of the electron
- g factor of the electron (relates magnetic dipole moment to the spin)
- possible future mass standard
- storage and control of charged particles (eg e^+ and $\bar{p}$ for antihydrogen production)

Magnetic field



Quiz: what is the sign of the charge q ?

For an electron in a field $|\vec{B}| =$

For a $|\vec{B}|$ field of $\approx 0.1 \text{ T}$:

an electron has $\nu_c \approx 164 \text{ GHz}$

(microwave regime)

a proton has $\nu_c \approx 89 \text{ MHz}$

(WM4J Gospel Music
from Oolitic IN ...)

Classically, the particle moves on the surface of a cylinder with axis parallel to $\vec{B}$.

The particle moves uniformly along the direction of $\vec{B}$; another field is needed to confine the charge fully.

Cyclotron frequency is

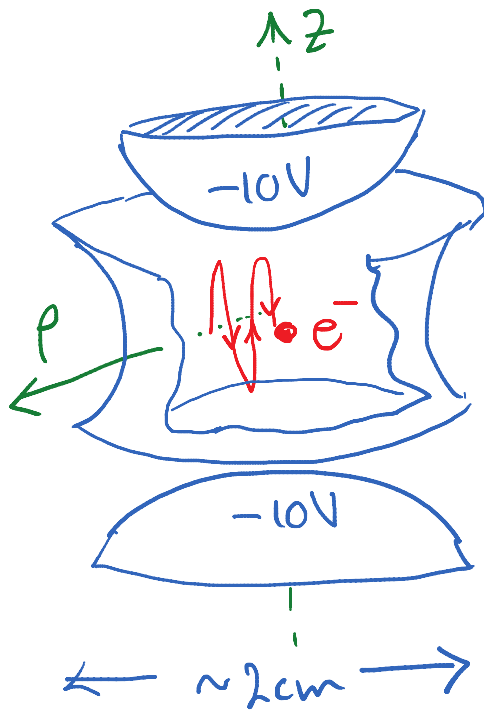
$$\omega_c = \frac{|qB|}{m}$$

Quadruple electric field

The confinement in the z direction is done by electrostatic repulsion.

Potential is: $V(\rho, z) = \frac{V_0}{2d^2} (z^2 - \frac{1}{2}\rho^2)$

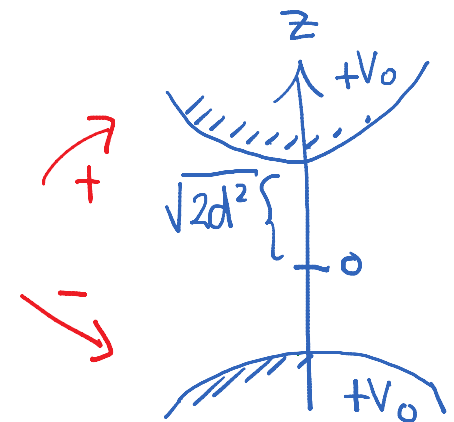
Ex. Show that $\vec{\nabla}^2 V = 0$
(Laplace's eq.)



The equipotential surfaces are hyperbolas of revolution

For example, when $V = -10V \equiv V_0$

$$\Rightarrow z = \pm \sqrt{2d^2 + \frac{1}{2}\rho^2}$$



Ex: Find $\vec{E} = -\vec{\nabla} V$

Exercise

i) Show that V satisfies Laplace's equation

$$V(\rho, z) = \frac{V_0}{2d} \left(z^2 - \frac{1}{2} \rho^2 \right)$$

Solution :

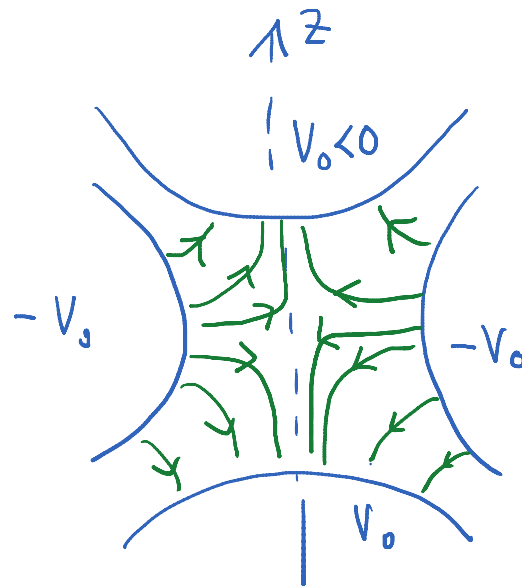
$$\begin{aligned} \nabla^2 V &= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \frac{V_0}{2d} \left(z^2 - \frac{1}{2} x^2 - \frac{1}{2} y^2 \right) \\ &= \frac{V_0}{2d} \left(-1 - 1 + 2 \right) = 0 \quad \checkmark \end{aligned}$$

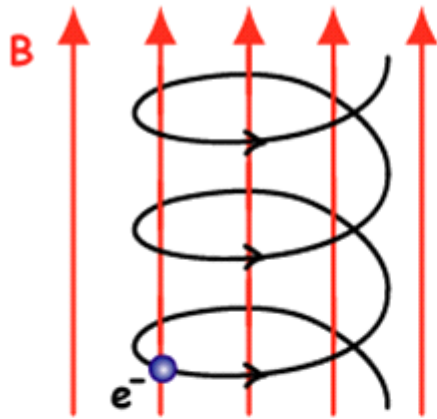
Exercise

Find $\vec{E} = -\vec{\nabla} V$ for $V = \frac{V_0}{2d^2} (z^2 - \frac{1}{2} \rho^2)$

$$\begin{aligned} \vec{E} &= -\frac{V_0}{2d^2} \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) \left(z^2 - \frac{1}{2} x^2 - \frac{1}{2} y^2 \right) \\ &= -\frac{V_0}{2d^2} \left[\hat{x}(-x) + \hat{y}(-y) + \hat{z}(2z) \right] = -\frac{V_0}{2d^2} (2z\hat{z} - x\hat{x} - y\hat{y}) \\ &= -\frac{V_0}{2d^2} (3z\hat{z} - \vec{r}) \end{aligned}$$

"Diagram"

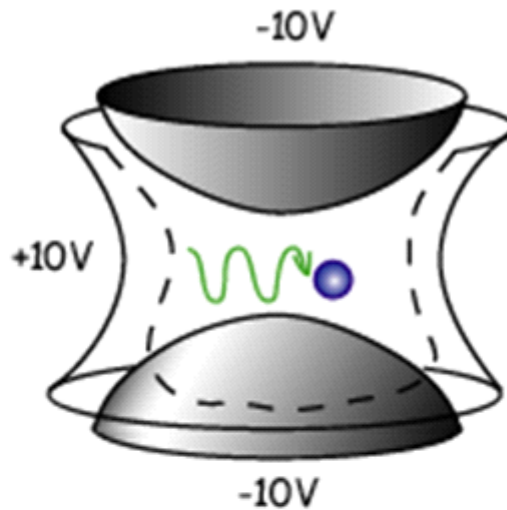




lateral confinement

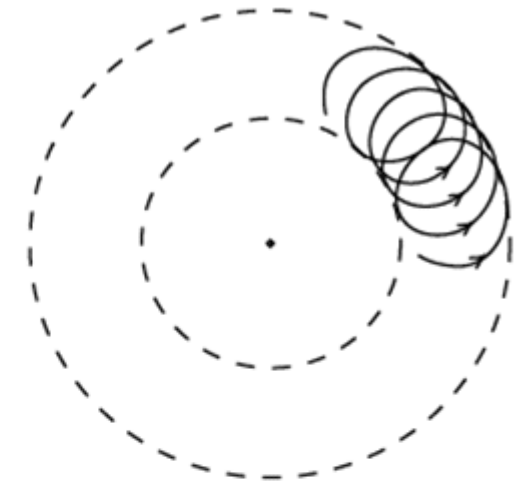
$$\omega_c = \left| \frac{qB}{m} \right|$$

$$\omega'_c = (\omega_c + \Omega) / 2$$



axial confinement

$$\omega_z = \sqrt{\frac{qV}{md^2}}$$



magnetron motion

$$\omega_m = (\omega_c - \Omega) / 2$$

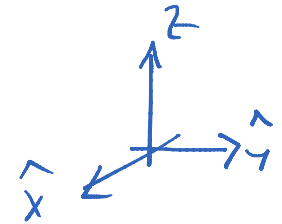
$$\Omega = (\omega_c^2 - 2\omega_z^2)^{1/2}$$

A review: Brown + Gabrielse RMP 58, 233 (1986)

Harmonic motion in axial direction

Classical motion of the charged particle

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad q\vec{E} + q(\vec{v} \times \vec{B}) = m\vec{\dot{v}}$$



$$-\frac{qV_0}{2d^2} (2z\hat{z} - x\hat{x} - y\hat{y}) + q(\dot{x}\hat{y} + \dot{y}\hat{x}) = m(\ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z})$$

$$\vec{v} \times \vec{B} = (\dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}) \times B\hat{z}$$

$$= -\dot{x}(-\hat{y}) + \dot{y}\hat{x} = \dot{x}\hat{y} + \dot{y}\hat{x}$$

The z -equation is decoupled from x and y :

$$-\frac{qV_0}{d^2} z = m\ddot{z}$$

Provided $qV_0 > 0$ (ie the endcaps repel the charge), the z motion is simple harmonic motion with frequency

$$\omega_z = \sqrt{\frac{qV_0}{md^2}}$$

For $V_0 = -10V$, $d = 3mm$, electron has

$$\nu_z = \omega_z/2\pi = 62 MHz \rightarrow \text{radio freq.}$$

The cyclotron frequency is about 1000x greater.

Quantum motion in Penning trap

The hyperbolic Penning trap can be solved analytically in

→ classical mechanics

(Newton's 2nd law)

→ nonrelativistic QM

(Schrödinger's equation)

These frameworks have limitations; for one thing spin is missing...

To study Lorentz violation in Penning traps using the Standard-Model Extension, we need to work in the framework of relativistic quantum mechanics. (ie the Dirac equation).

Dirac-equation analytical solution is known for the charged fermion in a uniform $\vec{B}$ field. [the Landau problem]

Since the electric field has a smaller (10^{-3}) impact on the energies, this will suffice to understand possible Lorentz violations

Plan: Find Landau solutions $\chi_{n,1s}^q$ in $\vec{B}$ field → use perturbation theory to find energy correction → seek experimental signals

Outlining the Landau solutions

Dirac equation :
$$(i \gamma^\mu D_\mu - m) \psi = 0$$

In this expression, have
$$D_\mu \equiv \partial_\mu + iqA_\mu$$

with A_μ containing the information about the $\vec{B}$ field.
We want $\vec{B} = B\hat{z}$ so may select any $\vec{A}$ such that $\vec{B} = \vec{\nabla} \times \vec{A}$.
Take, for example

$$\vec{A} = -By\hat{x}$$

Exercise: verify $\vec{B} = B\hat{z}$.

Then

$$A_\mu = (V, -\vec{A}) = (0, By, 0, 0)$$

Also: γ^μ are the 4 Dirac gamma matrices.

In the Dirac representation, we have 2×2 block form

$$\gamma^0 = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix} \quad \gamma^0 \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The spinor ψ is a 4-component thing

$$\psi = \begin{pmatrix} \sim \\ \sim \\ \sim \\ \sim \end{pmatrix}$$

Solutions

The solutions are quantized and we need two quantum #s :

$$S = \pm 1 \quad \text{"spin"}$$

$$n = 0, 1, 2, \dots \quad \text{principal quantum \#}$$

The solutions also depend on the two possible signs of

Energy quantization: $E \equiv qB$ For $\vec{B}$ along $\hat{z}$, $E < 0$ for electron.

$$(E_{n,S}^E)^2 = m^2 + p_z^2 + |qB|(2n+1 - SE)$$

Note i) $p_z = \text{constant}$: uniform drift along $\vec{B}$ direction.

ii) There are negative energies (positron interpretation)

iii) For the electron, the minimum of E^2 is $E^2 = m^2 + p_z^2$, and this occurs when $0 = 2n+1 - SE = 2n+1 + S \Rightarrow S = -1$
 $\Rightarrow$ lowest energy for the electron has spin opposite $\vec{B}$.

For proton, lowest energy state has $\uparrow_{\vec{S}} \uparrow_{\vec{B}}$ same directions.

Spinors. For example, the ground state for the electron is :

$$\psi_{n=0, s=-1}^{\epsilon < 0}(x^\mu) = C e^{i(p_x x + p_z z - E t)} e^{-(p_x + qBy)^2 / |qB|} \begin{bmatrix} 0 \\ E+m \\ 0 \\ -p_z \end{bmatrix}$$

complex

spin ↓

where

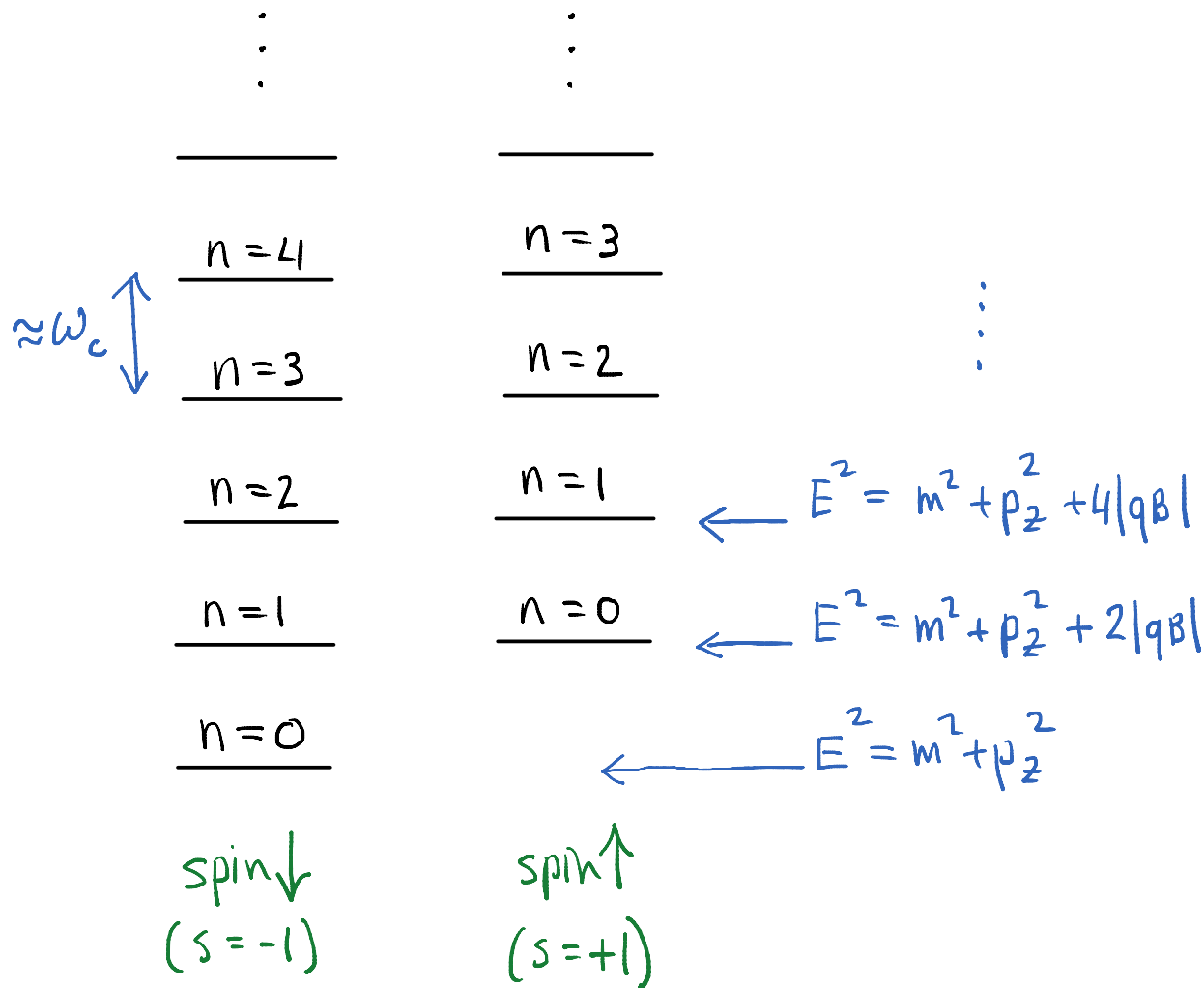
$$C = C_{n=0, s=-1}^{\epsilon < 0} = \left[\frac{\sqrt{|qB|}}{2\sqrt{\pi} L^2 E(E+m)} \right]^{\frac{1}{2}}$$

(and E should be $E_{n=0, s=-1}^{\epsilon < 0}$).

↙ $L =$ box size for x and z direction to normalize.

The spinors for higher n have Hermite polynomials $H_n(\dots)$ that come from solving the diff. eq. for the simple H. oscill.

Electron



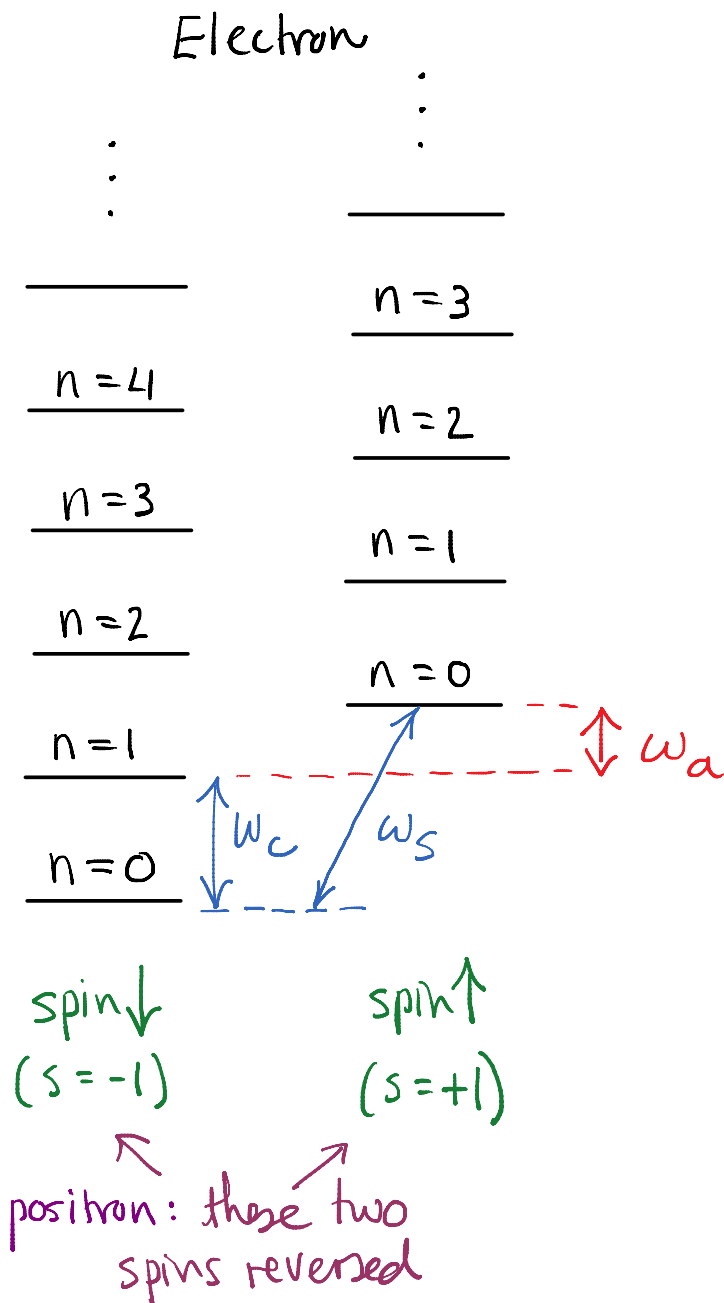
Note the two-fold degeneracy in $|n, s\rangle$ states above "ground:"

$$|2, -1\rangle \leftrightarrow |1, +1\rangle$$

$$|1, -1\rangle \leftrightarrow |0, +1\rangle$$

This degeneracy is broken by the anomalous magnetic moment of the electron $g \approx 2$ (not exact):

Anomalous magnetic moment



Dipole moment of e^- : $\vec{\mu} = g \mu_B \frac{1}{2} \vec{\sigma}$

where $\mu_B =$ Bohr magneton $= \frac{e\hbar}{2mc}$

$$H = -\vec{\mu} \cdot \vec{B} = -\frac{g}{2} \mu_B \vec{\sigma} \cdot \vec{B}$$

$$= -\frac{g}{2} \frac{1}{2} \frac{eB}{m} \sigma_3 = -\frac{g}{2} \frac{1}{2} \omega_c \sigma_3$$

$\Rightarrow$ Separation between $\uparrow$ and $\downarrow$ is

$$\omega_s = \frac{g}{2} \omega_c$$

Define anomaly frequency ω_a

$$\omega_a \equiv \omega_s - \omega_c$$

This can be experimentally measured; the energy in the transition is:

$$\hbar \omega_a = E_{0,+1}^{<0} - E_{1,-1}^{<0}$$

Also note

$$\hbar \omega_c = E_{1,-1}^{<0} - E_{0,-1}^{<0}$$

Lorentz violation in the Penning trap

To study Lorentz violation in Penning traps using the Standard-Model Extension, we start from the modified hamiltonian

$$i \partial_t \chi^q = (\hat{H}_0^q + \hat{H}_{\text{pert}}^q) \chi^q$$

(Chuck's talk)

$\hat{H}_0^q$ = CPT and Lorentz preserving hamiltonian

$$\begin{aligned} \hat{H}_{\text{pert}}^q = & a_\mu \gamma^0 \gamma^\mu - b_\mu \gamma_5 \gamma^0 \gamma^\mu - c_{00} m \gamma^0 - i(c_{0j} + c_{j0}) D^j + i(c_{00} D_j - c_{jk} D^k) \gamma^0 \gamma^j \\ & - d_{j0} m \gamma_5 \gamma^j + i(d_{0j} + d_{j0}) D^j \gamma_5 + i(d_{00} D_j - d_{jk} D^k) \gamma^0 \gamma_5 \gamma^j + \frac{1}{2} H_{\mu\nu} \gamma^0 \sigma^{\mu\nu}. \end{aligned}$$

If χ^q are solutions for $\hat{H}_{\text{pert}}^q = 0$, the corrections to the energies for the perturbative case are found from

$$\delta E_{n,s}^q = \int \chi_{n,s}^{q\dagger} \hat{H}_{\text{pert}}^q \chi_{n,s}^q d^3r$$

For the antiparticle, use the spinors $\bar{\chi}_{n,s}^q$ and note that the hamiltonian is found from charge conjugation (except we don't reverse $\vec{B}$).

Charge conjugation

$\hat{H}_{\text{pert}}^{\bar{q}}$ can be found from $\hat{H}_{\text{pert}}^q$ using the rules found in the Data Tables for Lorentz violation, Table XXVI:

Kostelecký, NR, RMP 83, 11 (2011) [arXiv:0801.0287]

Table XXVI. C, P, T properties of operators for Lorentz violation in QED

Coefficient	C	P	T	CP	CT	PT	CPT
$c_{TT}, c_{JK}, (k_F)_{TJTK}, (k_F)_{JKLM}$	+	+	+	+	+	+	+
$b_J, g_{JTL}, g_{JKT}, (k_{AF})_J$	+	+	-	+	-	-	-
$b_T, g_{JTT}, g_{JKL}, (k_{AF})_T$	+	-	+	-	+	-	-
$c_{TJ}, c_{JT}, (k_F)_{TJKL}$	+	-	-	-	-	+	+
a_T, e_T, f_J	-	+	+	-	-	+	-
H_{JK}, d_{TJ}, d_{JT}	-	+	-	-	+	-	+
H_{TJ}, d_{TT}, d_{JK}	-	-	+	+	-	-	+
a_J, e_J, f_T	-	-	-	+	+	+	-

Reverse the signs for $a_\mu, d_{\mu\nu}, H_{\mu\nu}, e_\mu, f_\mu$.

Energy shifts found

For the electron, the leading-order energy corrections are:

$$\begin{aligned}\delta E_{n,\pm 1}^{e-} &\approx \textcircled{+} a_0^e \mp b_3^e - c_{00}^e m_e \textcircled{\pm} d_{30}^e m_e \textcircled{\pm} H_{12}^e \\ &\quad - \frac{1}{2} (c_{00}^e + c_{11}^e + c_{22}^e) (2n + 1 \pm 1) \omega_c \\ &\quad - \left(\frac{1}{2} c_{00}^e + c_{33}^e \textcircled{\mp} d_{03}^e \textcircled{\mp} d_{30}^e \right) \frac{p_z^2}{m_e}\end{aligned}$$

Positron result follows by reversing the circled signs.

Shifts in measured quantities:

$$\omega_c^{e<0} \equiv E_{1,-1}^{e<0} - E_{0,-1}^{e<0} = (1 - c_{00}^e - c_{11}^e - c_{22}^e) \omega_c$$

Exercise: find

$$\omega_a^{e<0} \equiv E_{0,1}^{e<0} - E_{1,-1}^{e<0}$$

Solution

Exercise: find $\omega_a^{\epsilon < 0} \equiv E_{0,1}^{\epsilon < 0} - E_{1,-1}^{\epsilon < 0}$

Using... $\delta E_{n,\pm 1}^{e-} \approx a_0^e \mp b_3^e - c_{00}^e m_e \pm d_{30}^e m_e \pm H_{12}^e$
 $-\frac{1}{2}(c_{00}^e + c_{11}^e + c_{22}^e)(2n + 1 \pm 1)\omega_c$
 $-\left(\frac{1}{2}c_{00}^e + c_{33}^e \mp d_{03}^e \mp d_{30}^e\right) \frac{p_z^2}{m_e}$ we find...

$$\omega_a^- = E_{0,1}^- - E_{1,-1}^-$$

$$\approx \omega_a + \underbrace{\delta E_{0,+1}^- - \delta E_{1,-1}^-}_{\text{terms independent of spin will not contribute; so consider only ones with } \pm \text{ in front.}}$$

$$= \omega_a - b_3^e + d_{30}^e m_e + H_{12}^e + (d_{03}^e + d_{30}^e) p_z^2 / m_e$$

$$- b_3^e + d_{30}^e m_e + H_{12}^e + (d_{03}^e + d_{30}^e) p_z^2 / m_e$$

$$\omega_a^- \approx \omega_a - 2b_3^e + 2d_{30}^e m_e + 2H_{12}^e \quad \text{For electron.}$$

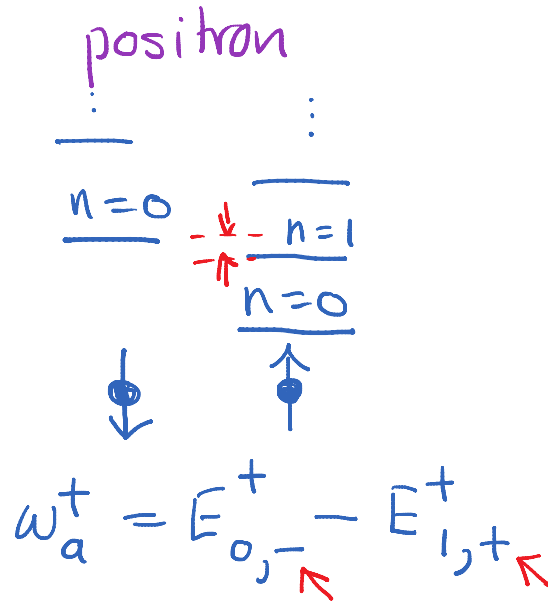
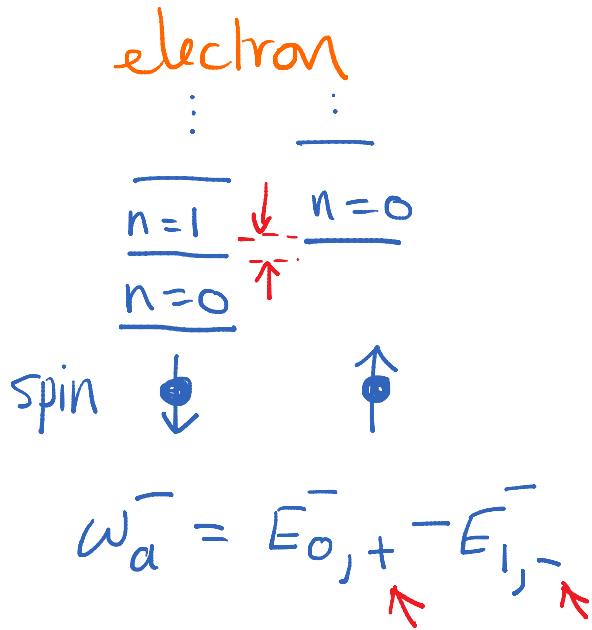
Exercise: find ω_a^+ for the positron.

Positron anomaly frequency

Two steps to get ω_a^+ from $\omega_a^- = \omega_a - 2b_3^e + 2d_{30}^e m + 2H_{12}^e$

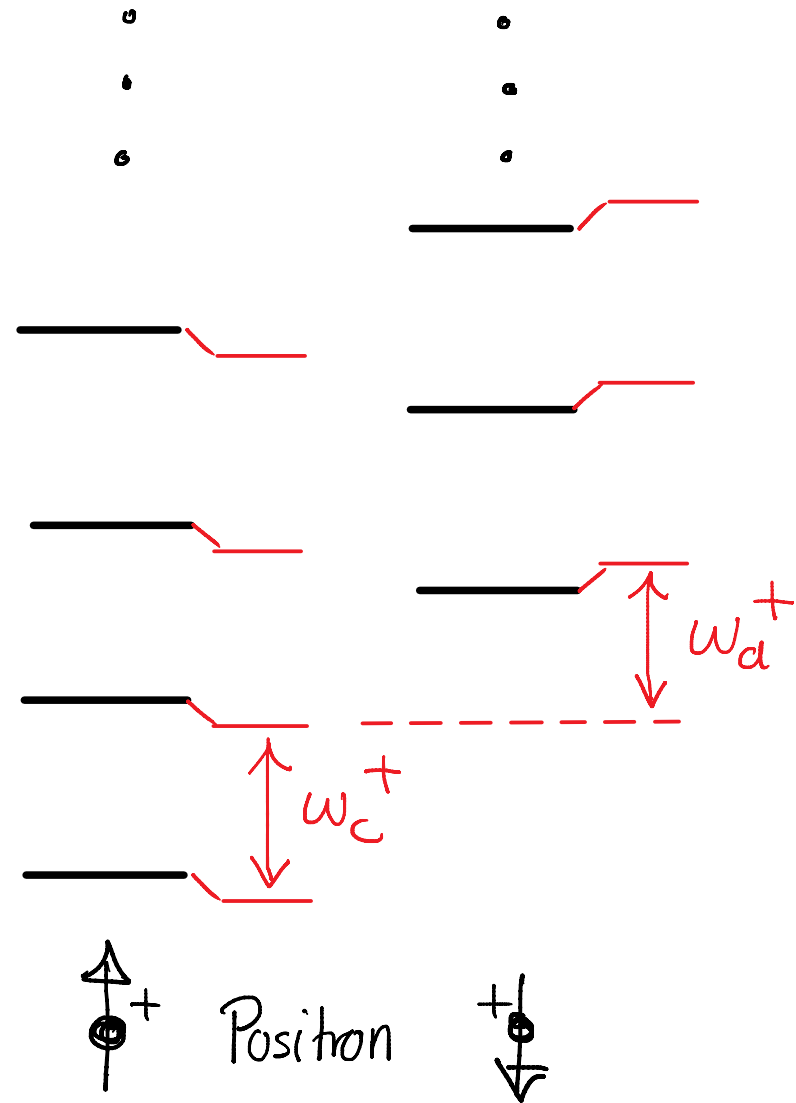
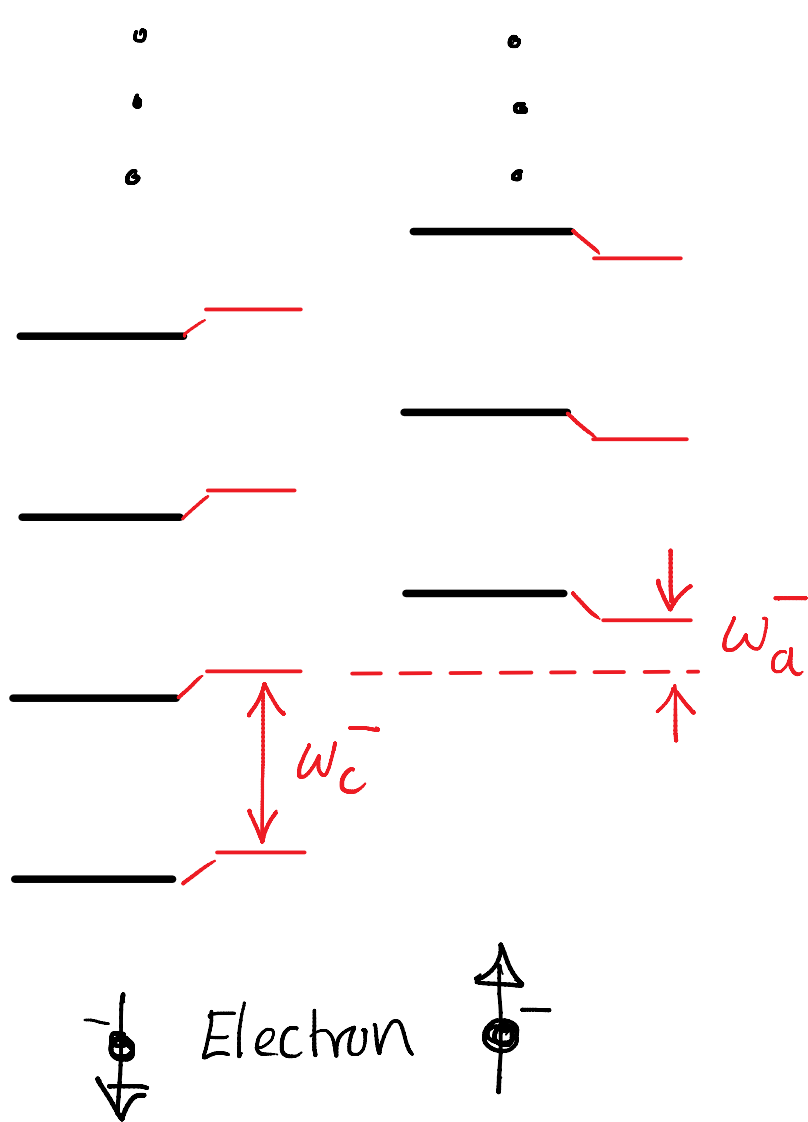
i) to get $\delta E_{n,\pm}^+$ reverse the signs of $a_\mu^e, d_{\mu\nu}^e, H_{\mu\nu}^e$ in $\delta E_{n,\pm}^-$

ii) Recall that ω_a^+ has opposite spins



$$\omega_a^{\pm} = \omega_a \pm 2b_3^e + 2d_{30}^e m + 2H_{12}^e$$

Energy-level diagrams for the b coefficient

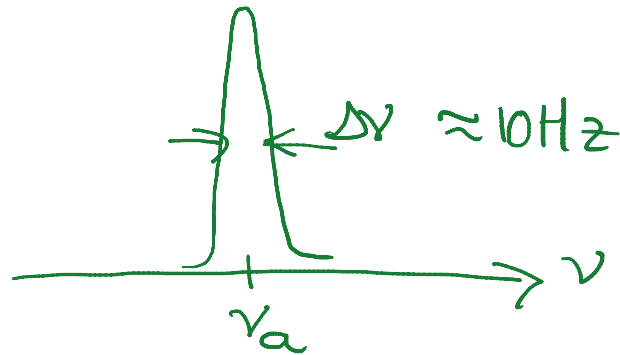


$$\omega_a^{\pm} = \omega_a \pm 2b_3^e + 2d_{30}^e m + 2H_{12}^e$$

$$\omega_c^{\pm} = \omega_c - (c_{00} + c_{11} + c_{22}) \omega_c$$

Penning-trap limits on b coefficient

Find the limit on b_3^e in an experiment comparing ν_a^- to ν_a^+ assuming each frequency is resolved at the level of $\Delta\nu \approx 10\text{Hz}$.



In the absence of a signal, we may conclude that

$$|\nu_a^- - \nu_a^+| \lesssim 10\text{Hz}$$

$$\text{but } 2\pi(\nu_a^- - \nu_a^+) \approx (\omega_a - 2b_3^e) - (\omega_a + 2b_3^e) = -4b_3^e$$

$$\Rightarrow 4|b_3^e| \lesssim 2\pi(10\text{Hz})\hbar \quad (\text{replace } \hbar = 1)$$

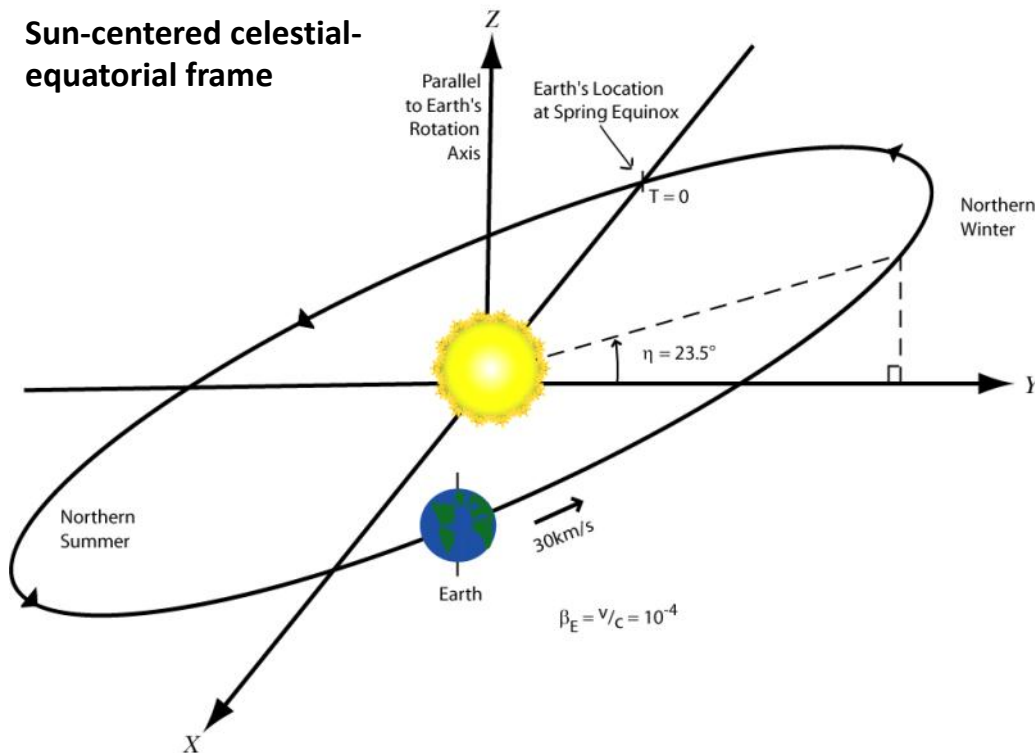
$$\Rightarrow |b_3^e| < \frac{1}{2}\pi(10\text{s}^{-1})(7 \times 10^{-25}\text{GeV} \cdot \text{s}) \approx 10^{-23}\text{GeV}$$

$$\Rightarrow |b_3^e| < 10^{-23}\text{GeV}$$

Instantaneous and sidereal signals

The result $|b_3^e| < 10^{-23} \text{ GeV}$ is done by comparing w_a^- , w_a^+ at the same time. The subscript 3 indicates the axis of the $\vec{B}$ field. Note that b_μ^e is fixed in the Sun-centered celestial equatorial reference frame, but $\vec{B}$ axis changes as the Earth rotates and orbits around the sun.

$\Rightarrow$ "instantaneous" tests must compare w_a^- , w_a^+ for identical $\vec{B}$ orientations relative to the Sun frame.



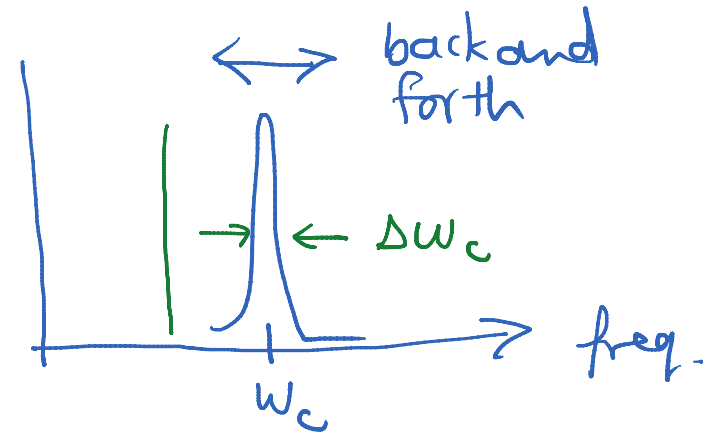
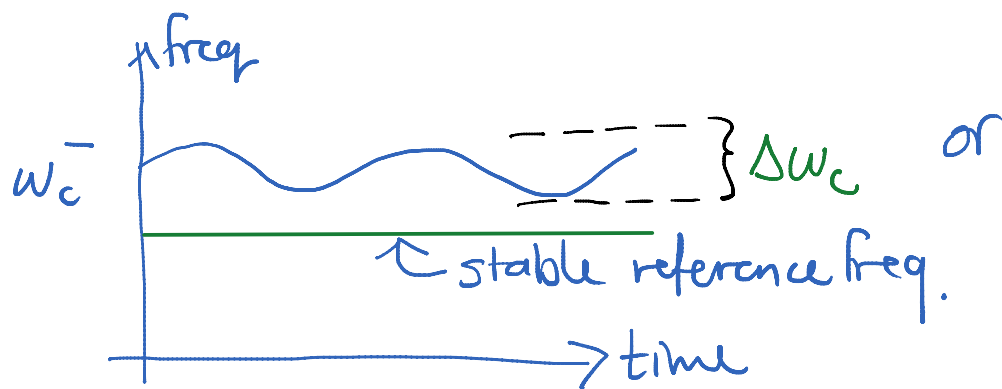
- Expts need to:
- swap e^+ , e^- quickly, and
 - measure w_a^+ , w_a^- over a short period.

Sidereal signals

The cyclotron frequencies for e^- and e^+ have identical shifts:

$$\omega_c^\pm = (1 - c_{00}^e - c_{11}^e - c_{22}^e) \omega_c$$

A way to measure or bound the $c_{\mu\nu}$ combination is to monitor ω_c^- over a long time and look for oscillations.



Exercise

Assuming no signal is detected and ω_c^- fluctuates at the level of 10^{-7} during the experiment, what bound is placed on the c 's?

Solution: estimate c limit

Exercise

Assuming no signal is detected and ω_c^- fluctuates at the level of 10^{-7} during the experiment, what bound is placed on the c's?

We have $\omega_c^- = (1 - c_{00} - c_{11} - c_{22}) \omega_c$

Variations must be due to the spatial components since $c_{00} = \text{constant in time}$.

$$\Rightarrow \Delta \omega_c^- = |c_{11} + c_{22}| \omega_c$$

$$\Rightarrow \Delta \omega_c^- / \omega_c = |c_{11} + c_{22}|$$

We are given that

$$\frac{\Delta \omega_c^-}{\omega_c} < 10^{-7}$$

so it follows that

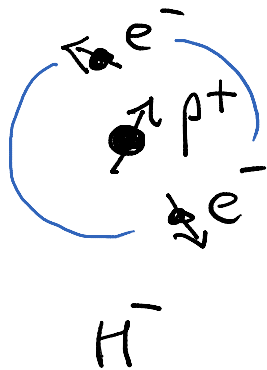
$$|c_{11}^e + c_{22}^e| < 10^{-7}$$

Other Penning-trap tests

1. Proton-antiproton comparisons, to place limits on b_3^p , $d_{\mu\nu}^p$, $H_{\mu\nu}^p$, $c_{\mu\nu}^p$ etc
Harder to do than with electron because need $\bar{p}$ supply, and frequencies are lower.

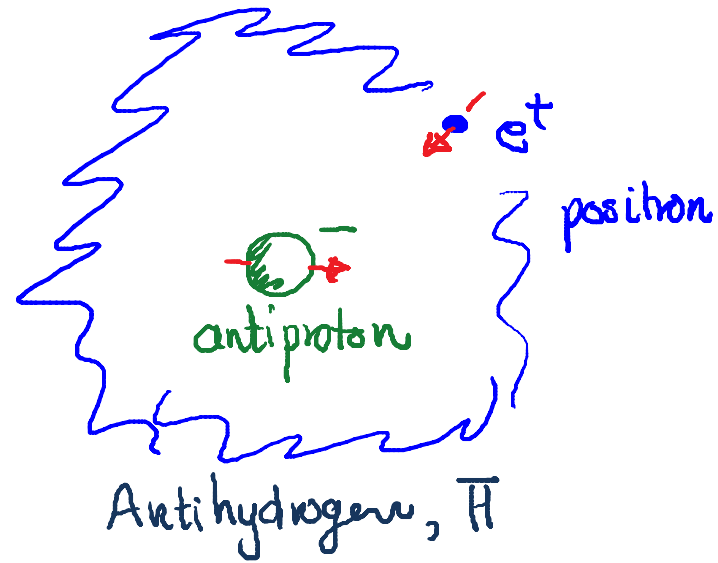
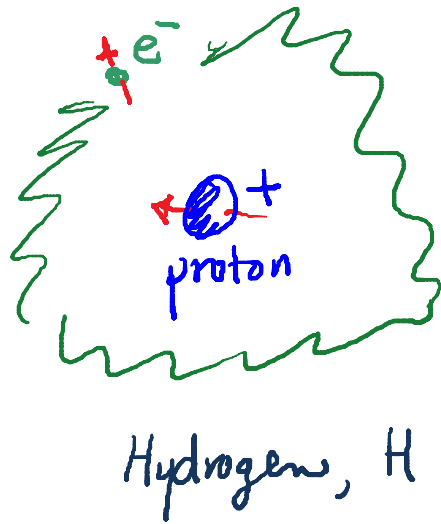
2. H^- ion compared with $\bar{p}$.

Point: same charge, so easier to swap them.



→ some results for $c_{\mu\nu}$ coefficients.

H and antihydrogen



How to make $\bar{H}$?

Where are $\bar{H}$ experiments being done?

How do the SME fields affect the spectra of H, $\bar{H}$?

Which transitions have leading-order SME effects?

Some answers : Bluhm, Kostelecký, NR, PRL 82, 2254 (1999)

Source of $\bar{p}$ is needed - CERN, Fermilab

1. Free antihydrogen

1996: eleven $\bar{H}$ atoms reported (PLB 368, 251) by LEAR

1998: Fermilab - beam of $\bar{H}$ (PRL 80, 3037) - Blanford et al

2. Trapped antihydrogen

ATRAP group (Gabrielse)

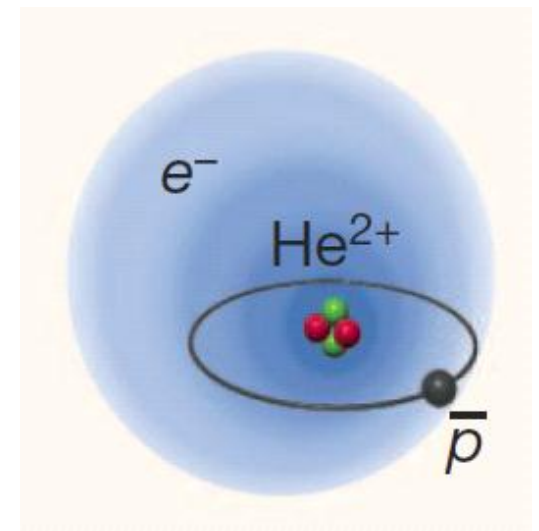
ALPHA group, formerly ATHENA

} both at CERN

3. Also CERN-based.

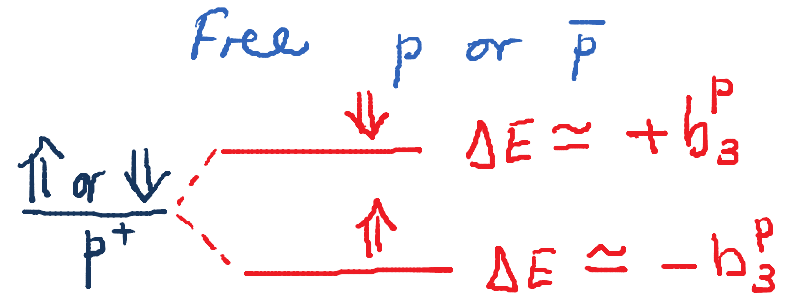
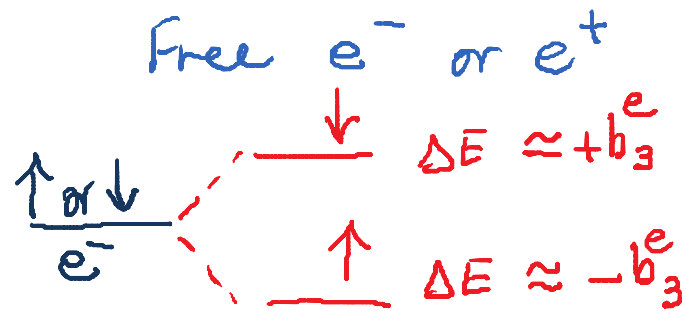
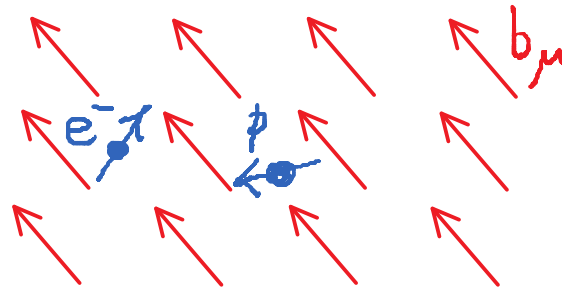
ASACUSA - Antiprotonic Helium spectroscopy

AEgIS - investigating gravitational accel.
of $\bar{H}$



Free hydrogen and antihydrogen

What effect does the b_μ term have on H and $\bar{H}$ in the absence of external $\vec{E}$ or $\vec{B}$ fields?



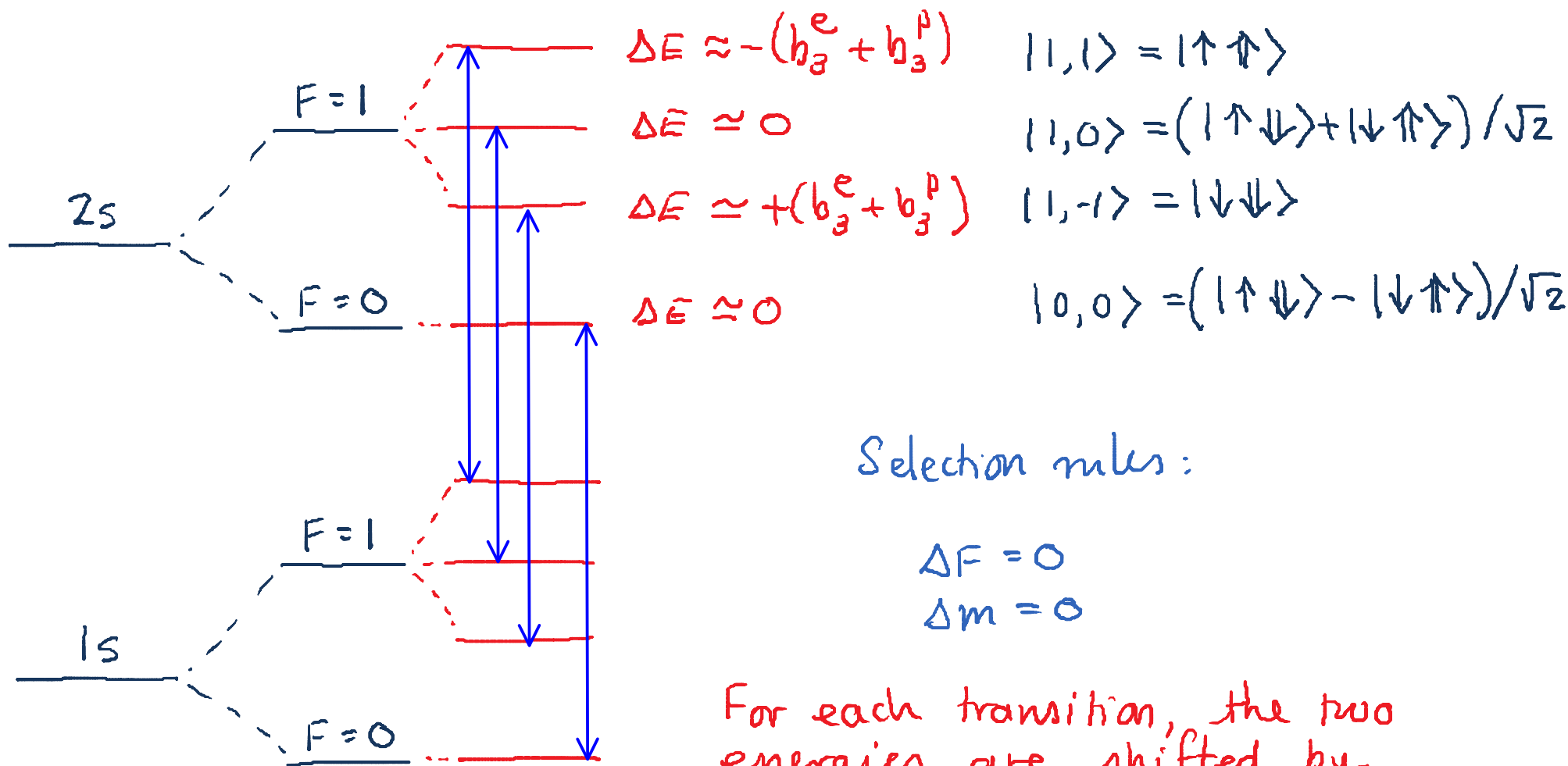
These are leading-order corrections.

Terms due to Coulomb interaction will be suppressed by $(qV)/m_p \ll 1$.

Recall that for the antiparticles, the sign of $b_\mu \rightarrow +b_\mu$ in the Hamiltonian, due to charge conjugation.

1S to 2S spectroscopy of free H and anti H

The unbound states couple together forming a singlet and a triplet. These are $|F, m_F\rangle$ with $|0, 0\rangle$ and $|1, 1\rangle, |1, 0\rangle, |1, -1\rangle$.



Selection rules:

$$\Delta F = 0$$

$$\Delta m = 0$$

(Similar result holds for $a_\mu, c_{\mu\nu}, d_{\mu\nu}, H_{\mu\nu}$)

For each transition, the two energies are shifted by the same amount. So, no leading-order shift occurs in 1S to 2S transitions of free H or $\bar{H}$.

Trapping neutral atoms

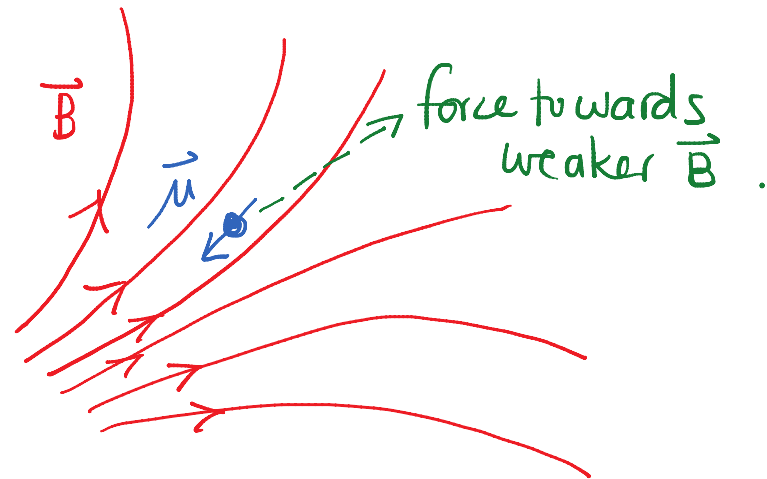
Since H and $\bar{H}$ are neutral, they cannot be trapped electrostatically.
 $\bar{H}$ can be confined using its dipole moment.

Trapping requires a bowl-shaped potential - weakest $|\vec{B}|$ at center

$$U = -\vec{\mu} \cdot \vec{B}$$

$$= \pm g \mu_B \vec{S} \cdot \vec{B}$$

electron: $\vec{\mu}$ is opposite $\vec{S}$
positron: $\vec{\mu}$ and $\vec{S}$ parallel.



Dipole moment and spin:

$$\vec{\mu} = \pm g \mu_B \vec{S}$$

$$\mu_B = e\hbar/2m_e$$

Bohr magneton

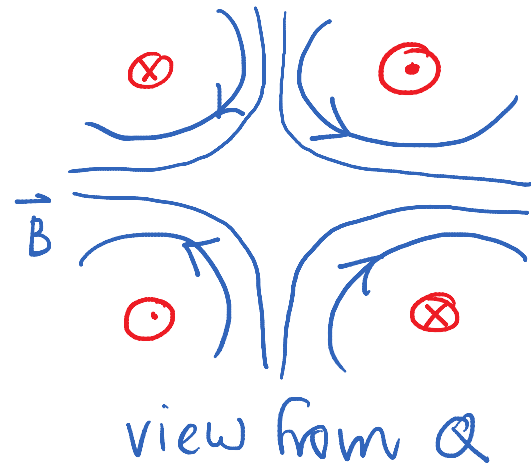
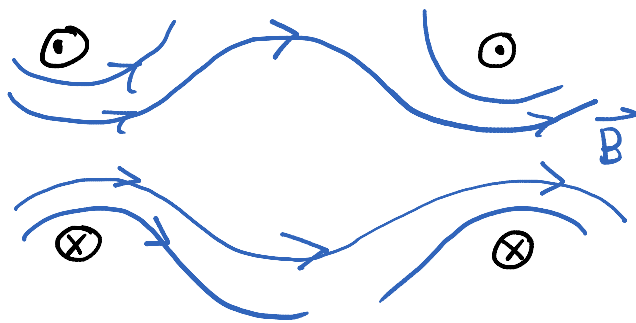
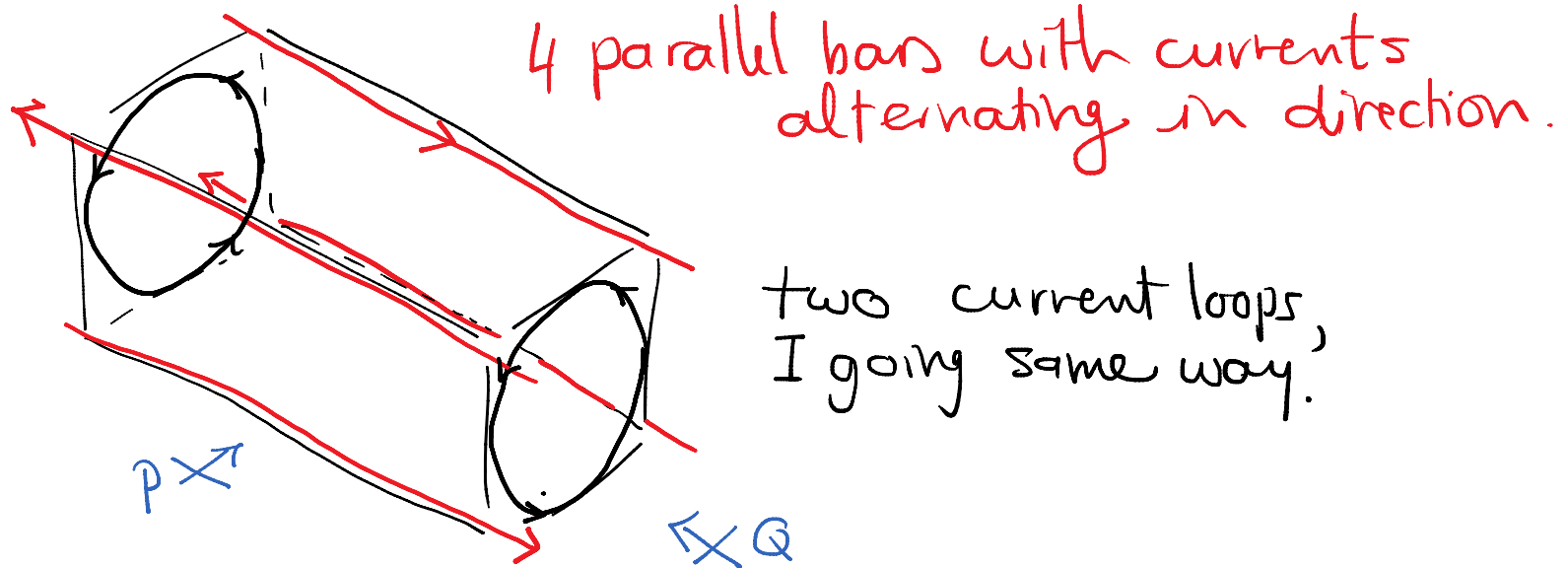
$$\mu_N = e\hbar/2m_p \approx 10^{-3} \mu_B$$

Nuclear magneton

⇒ In $\bar{H}$, interaction with the e^+ dominates

$$U \approx -g \mu_B \vec{S} \cdot \vec{B} \Rightarrow s = \downarrow \text{ is trapped (weak-field seeking)}$$

Ioffe-Pritchard Trap for neutral particles



Quadrupole $\vec{B}$

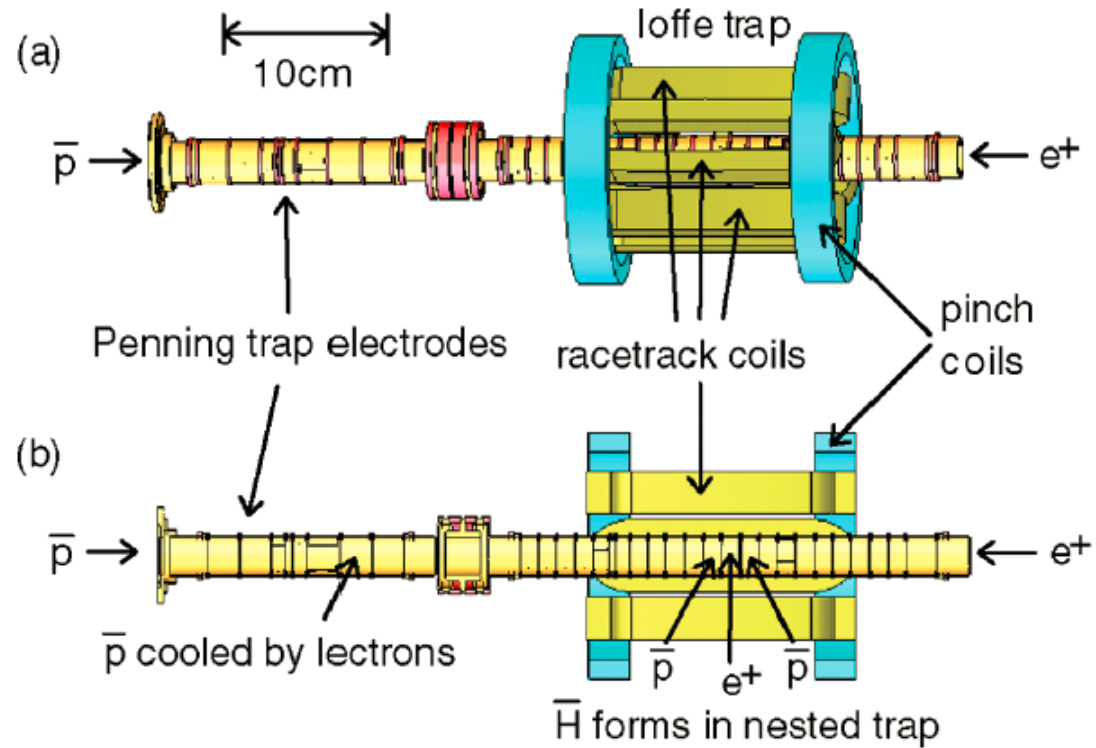
Point is to create a region of weakest $|\vec{B}|$ to trap weak field seekers ($\vec{\mu}$ opposite to $\vec{B}$)

ATRAP apparatus

Penning trap: for charged particles

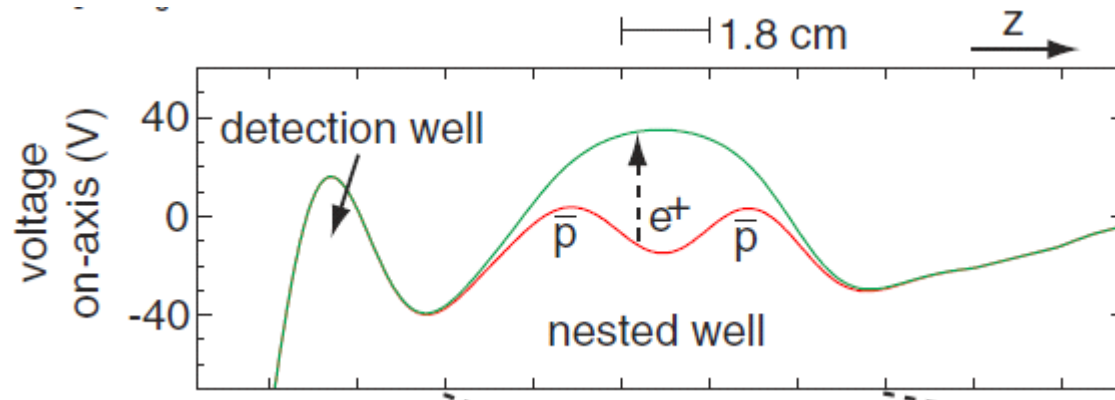
Ioffe-Pritchard trap: for neutral $\bar{H}$ confinement.

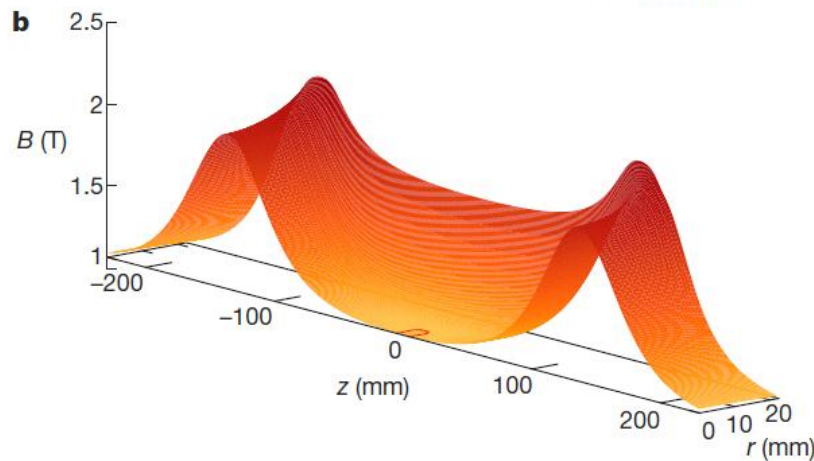
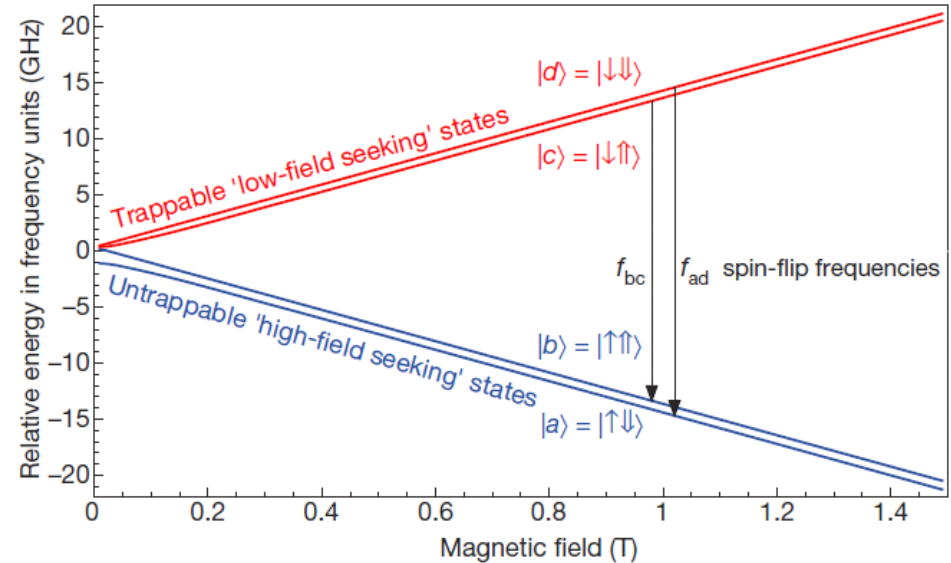
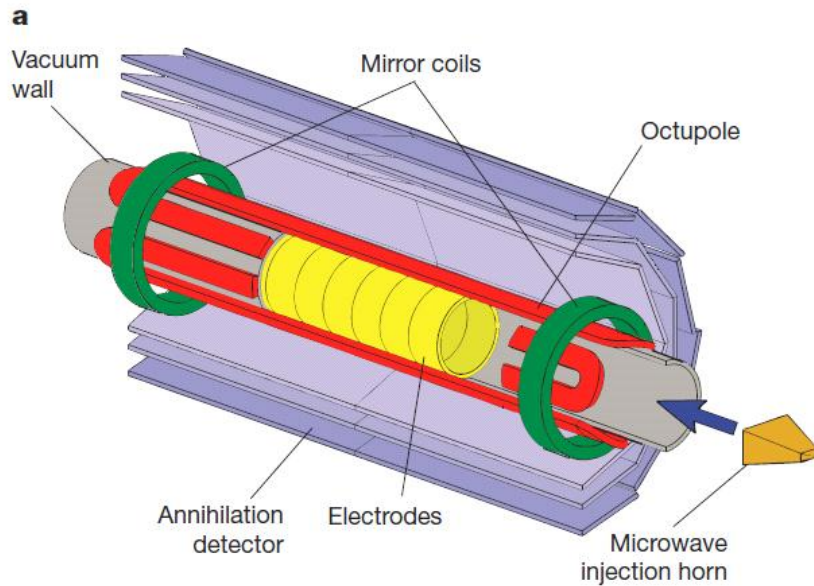
Not shown: solenoid used to create a uniform $\vec{B}$ field (1T) along the cylindrical axis.



Red: potential used to confine positrons (e^+) and antiprotons ($\bar{p}$).

Green: potential after $\bar{H}$ production.

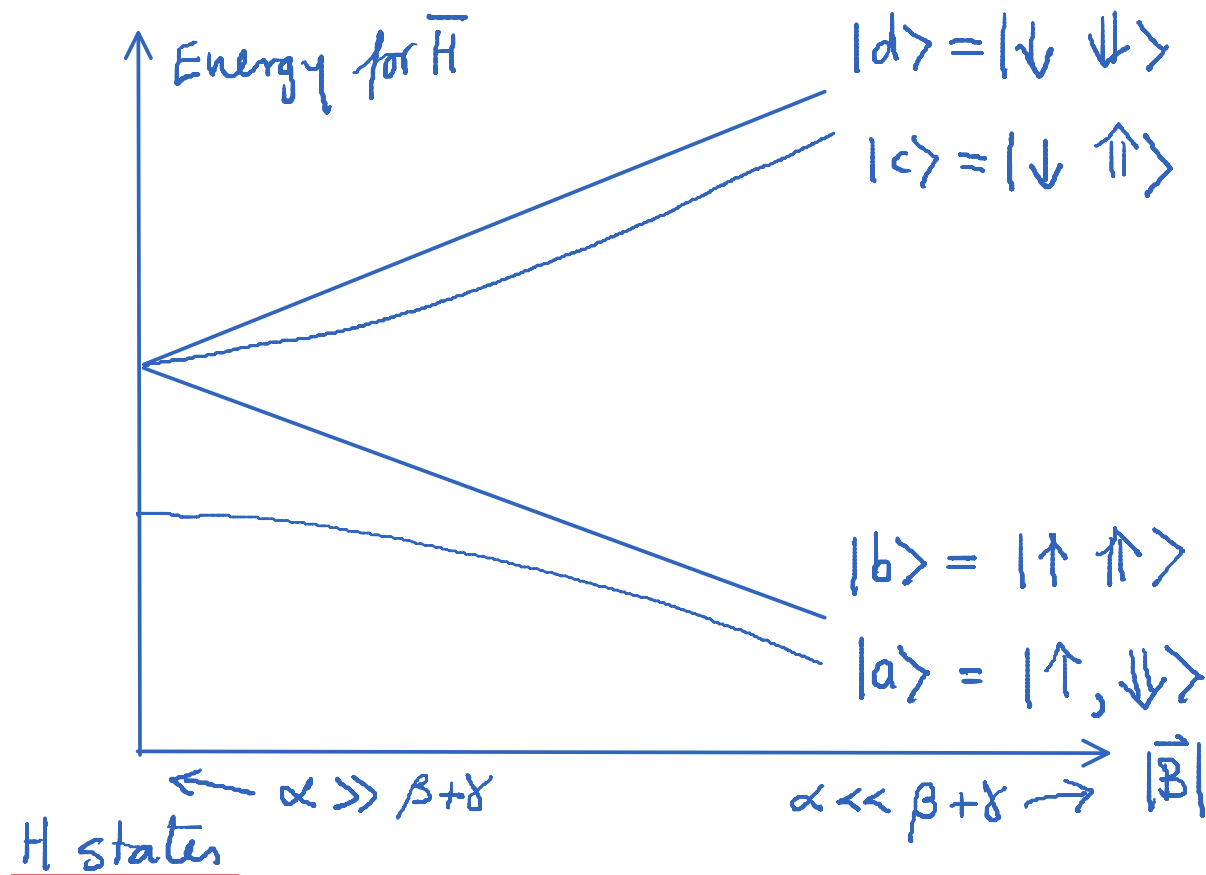




Used microwaves to induce spin flip of the electron. This changes a trapped (low-field seeking) state into an untrapped one. The untrapped states can be detected as they annihilate

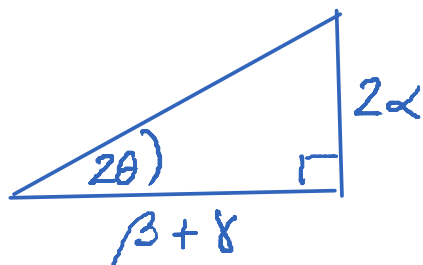
→ Thursday poster session: Joseph McKenna

spin $|s_{e^+}, s_{p^-}\rangle$



For H in the same $\vec{B}$ field, reverse the spins.

$$H^{\text{spins}} = \alpha \vec{\sigma}_e \cdot \vec{\sigma}_p + \beta \sigma_3^e - \gamma \sigma_3^p$$

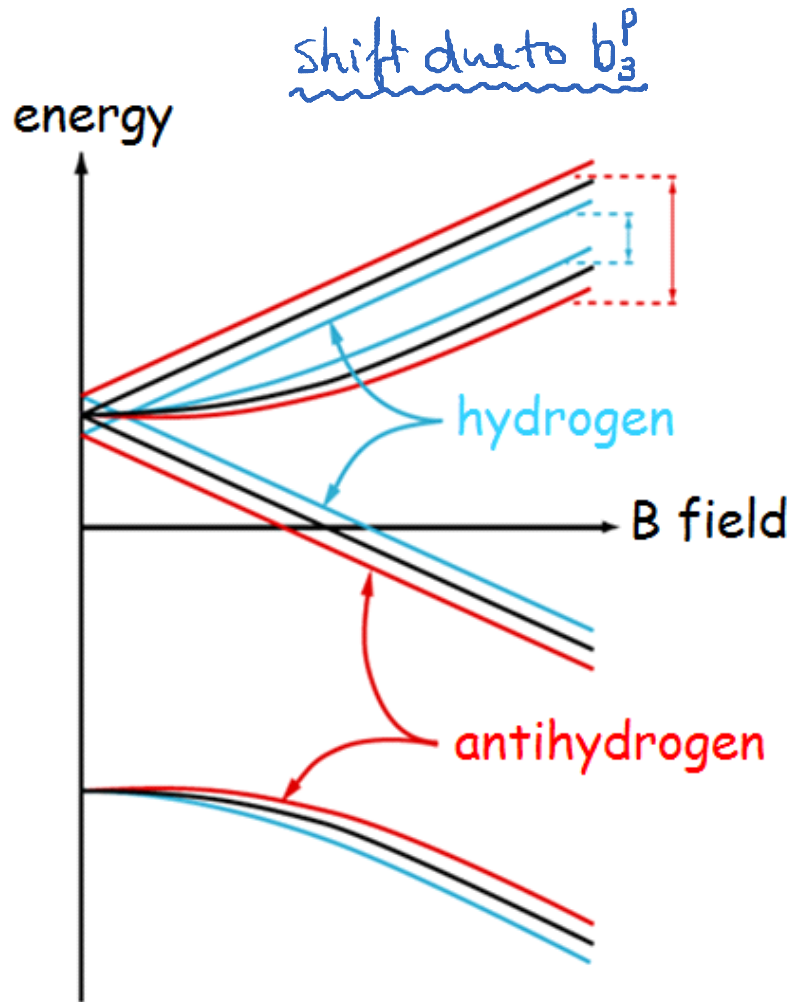


$$|d\rangle = |\uparrow \uparrow\rangle$$

$$|c\rangle = \sin\theta |\downarrow \uparrow\rangle + \cos\theta |\uparrow \downarrow\rangle$$

$$|b\rangle = |\downarrow \downarrow\rangle$$

$$|a\rangle = \cos\theta |\downarrow \uparrow\rangle - \sin\theta |\uparrow \downarrow\rangle$$



Shifts in the energies for b_μ :

$$\Delta E_d \cong -(b_3^e + b_3^p)$$

$$\Delta E_c \cong -(b_3^e - b_3^p) \cos 2\theta$$

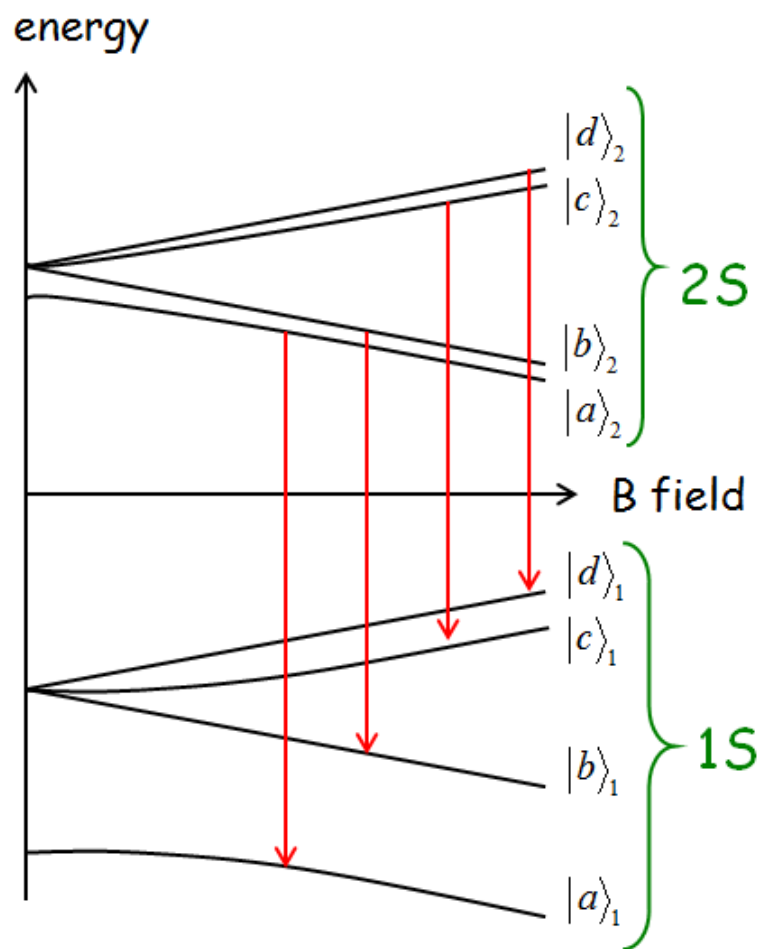
$$\Delta E_b \cong +(b_3^e + b_3^p)$$

$$\Delta E_a \cong +(b_3^e - b_3^p) \cos 2\theta$$

$$\left[\begin{array}{l} B \rightarrow 0 \Rightarrow \cos 2\theta \rightarrow 0 \\ B \rightarrow \infty \Rightarrow \cos 2\theta \rightarrow 1 \end{array} \right.$$

For $\bar{H}$, reverse the sign of b_μ .

1S to 2S in trapped antihydrogen?



- * $|d\rangle_2 \rightarrow |d\rangle_2$ has no leading-order signal because the spins are identical.
- * $|c\rangle_2 \rightarrow |c\rangle_1$ does have an unsuppressed signal, because the mixing angles Θ_n differ.
Optimum occurs for $|\vec{B}| \approx 0.01\text{T}$
Advantage over $d \rightarrow d$ is $\approx 10^5$.
- * However, this line is field dependent and no Zeeman broadening is a major limitation.

c to d hyperfine transition

1. The $|c\rangle$ to $|d\rangle$ hyperfine transition has a field-independent point

$$\nu_{c \rightarrow d} \text{ indep. of } |\vec{B}| \text{ at } |\vec{B}| \approx 0.65 \text{ T}$$

$\Rightarrow$ this line does not suffer from Zeeman broadening.

2. Furthermore, the transition has an unsuppressed signal for L.V.

$$\text{For } |c\rangle \rightarrow |d\rangle: \quad \delta(2\pi\nu^H) \cong -b_3^P - b_3^P = -2b_3^P$$

$$\text{and } \delta(2\pi\nu^{\bar{H}}) \cong +2b_3^P$$

ie when H frequency is increased by $2b_3^P$, the $\bar{H}$ frequency decreases.

Exercise. Consider an instantaneous $H \bar{H}$ comparison, $c \rightarrow d$.
If H resolution was at 1 MHz level, and no signal was seen, estimate the order of magnitude limit on b_3^P in GeV units.

$$\left[h = 7 \times 10^{-25} \text{ GeV} \cdot \text{s} \right]$$

Solution...

Consider an instantaneous $H \bar{H}$ comparison, $c \rightarrow d$.
If $\bar{H}$ resolution was at 1 mHz level, and no signal was seen, estimate the order of magnitude limit on b_3^P in GeV units.

Solution

(Replace $\hbar=1$ factor)

$$\omega_{c \rightarrow d}^H - \omega_{c \rightarrow d}^{\bar{H}} = -4b_3^P$$

$$4|b_3^P| < \underbrace{(2\pi)(10^{-3} \text{ s}^{-1})}_{\Delta\omega = 2\pi\Delta\nu} \hbar$$

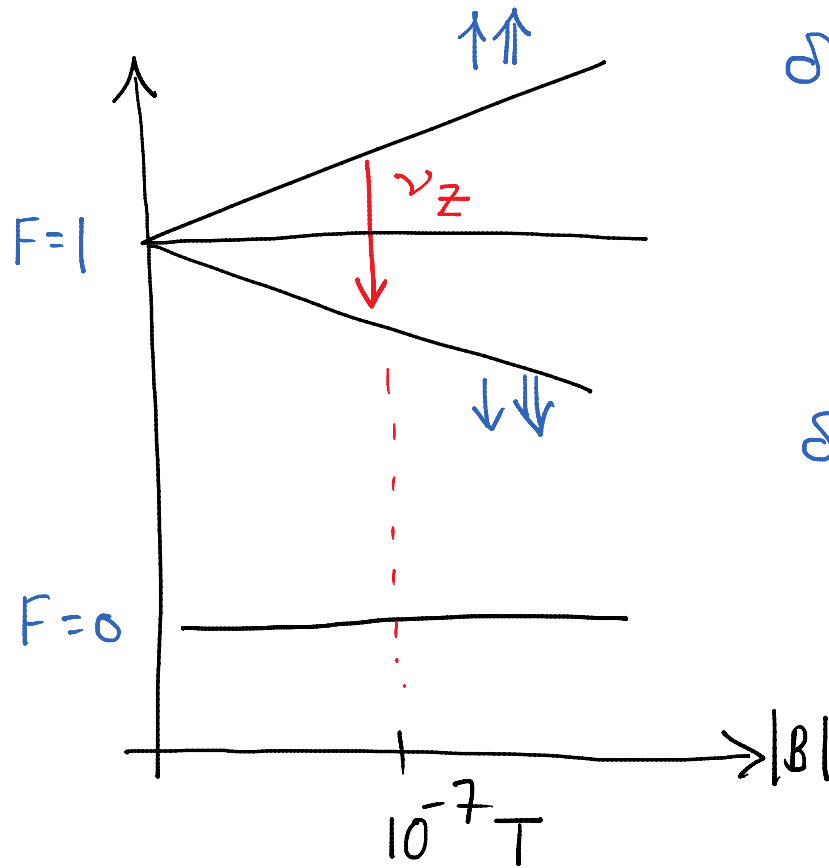
$$\Rightarrow |b_3^P| < \frac{1}{2}\pi \left(10^{-3} \frac{1}{\text{s}}\right) (7 \times 10^{-25} \text{ GeV s})$$

$$\Rightarrow |b_3^P| < 10^{-27} \text{ GeV} \quad \left(7 \times \frac{1}{2} \times \pi \approx 10\right)$$

H maser limits

Phillips et al., PRD 63, 111101 (2001)

H maser at Harvard-Smithsonian
Center for Astrophysics.



$$\begin{aligned} \delta E_{1,1} &= -b_3^e + d_{30}^e m_e + H_{12}^e \\ &\quad - b_3^p + d_{30}^p m_p + H_{12}^p \end{aligned}$$

$$\equiv -(\tilde{b}_3^e + \tilde{b}_3^p)$$

$$\delta E_{1,-1} = \tilde{b}_3^e + \tilde{b}_3^p$$

Shift in energy of ν_z line

$$\begin{aligned} 2\pi \delta \nu_z &= \delta E_{1,1} - \delta E_{1,-1} \\ &= -2(\tilde{b}_3^e + \tilde{b}_3^p) \end{aligned}$$

Phillips et al found: $\delta \nu_z < 0.4 \text{ mHz}$

$$\Rightarrow |\tilde{b}_3^e + \tilde{b}_3^p| = \pi \hbar \delta \nu_z \lesssim 3 \times (7 \times 10^{-25} \text{ GeV} \cdot \text{s}) (4 \times 10^{-4} \text{ s}^{-1})$$

$$\Rightarrow |\tilde{b}_3^e + \tilde{b}_3^p| < 10^{-27} \text{ GeV}$$

Finally

Not discussed here ...

- atomic-clock experiments on Earth, in space (Chuck)
- magnetometers, dual-species masers
- torsion pendulum experiments
- future analyses of these experiments for nonminimal SME (Matt...)
- ⋮

Discussed here

- Basic procedure for carrying out an SME analysis and deducing expected sensitivities
- Recurring theme: leading-order signals in fermions
 $\rightsquigarrow$ spin flips.

